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Mathematics Syllabus Advanced (3 Periods) – S6-S7

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Entry into force on 1 September 2021 for S6
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1st Baccalaureate session in June 2023

[1] Update of the oral record sheet: the BIS approved by means of Written Procedure 2022/60, the document "European Schools' Criteria for the Assessment of the Baccalaureate Oral Exams – Applicable as from European Baccalaureate session 2023" (Ref. 2022-09-D-46-en) on 5 December 2022 with an immediate entry into force.

[2] The syllabus has undergone minor changes, including clarifications, rewording, error corrections and harmonisation between the language versions.

European Schools Mathematics Syllabus Year S6&7 Advanced (3P)

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1. General Objectives

The European Schools have the two objectives of providing formal education and of encouraging pupils' personal development in a wider social and cultural context. Formal education involves the acquisition of competences (knowledge, skills and attitudes) across a range of domains. Personal development takes place in a variety of spiritual, moral, social and cultural contexts. It involves an awareness of appropriate behaviour, an understanding of the environment in which pupils live, and a development of their individual identity.

These two objectives are nurtured in the context of an enhanced awareness of the richness of European culture. Awareness and experience of a shared European life should lead pupils towards a greater respect for the traditions of each individual country and region in Europe, while developing and preserving their own national identities.

The pupils of the European Schools are future citizens of Europe and the world. As such, they need a range of competences if they are to meet the challenges of a rapidly changing world. In 2006 the European Council and European Parliament adopted a European Framework for Key Competences for Lifelong Learning. It identifies eight key competences which all individuals need for personal fulfilment and development, for active citizenship, for social inclusion and for employment:

1. Literacy competence
2. Multilingual competence
3. Mathematical competence and competence in science, technology and engineering
4. Digital competence
5. Personal, social and learning to learn competence
6. Citizenship competence
7. Entrepreneurship competence
8. Cultural awareness and expression competence

The European Schools' syllabuses seek to develop all of these key competences in the pupils.

Key competences are that general, that we do not mention them all the time in the Science and Mathematics syllabuses.

2. Didactical Principles

General

In the description of the learning objectives, competences, connected to content, play an important role. This position in the learning objectives reflects the importance of competences acquisition in actual education. Exploratory activities by pupils support this acquisition of competences, such as in experimenting, designing, searching for explanations and discussing with peers and teachers. In science education, a teaching approach is recommended that helps pupils to get acquainted with concepts by having them observe, investigate and explain phenomena, followed by the step to have them make abstractions and models. In mathematics education, investigations, making abstractions and modelling are equally important. In these approaches, it is essential that a maximum of activity by pupils themselves is stimulated – not to be confused with an absent teacher: teacher guidance is an essential contribution to targeted stimulation of pupils' activities.

Mathematics

Careful thought has been given to the content and the structure to where topics are first met in a pupil's time learning mathematics in secondary education. It is believed that this is a journey and if too much content is met at one point, there is a risk that it will not be adequately understood and thus a general mathematical concept will not be fully appreciated. By limiting the content of this syllabus (found in section 4.2.) each year more time can be used to develop core mathematical concepts that may have been met before or new mathematical concepts introduced are given ample time for extension. It must be noted that extension activities are conducted at the discretion of the teacher, however, it is suggested that rather than look at a vertical approach to extension a horizontal approach is used, thus giving the pupil a deeper understanding of the mathematical concept (in section 4 the word 'limitation' is used to ensure the extension does not go too far).

Furthermore, to this point it is believed that with a focus on competences this syllabus can encourage pupils to have a greater enjoyment of mathematics, as they not only understand the content better but understand the historical context (where it is expected a history of mathematics can be told over the cycles) and how the mathematics can be applied in other subjects, cross cutting (these can be seen in the fourth column in section 4.2.). As such the syllabuses have specifically been designed with reflection to the key competences (section 1) and the subject specific competences (section 3.1). In some cases, the key competences are clear for example the numerous history suggested activities (shown by the icon ) that maps to key competency 8 (Cultural awareness and expression). In other areas the link may not be so apparent.

One of the tasks in the pupil's learning process is developing inference skills, analytical skills and strategic thinking, which are linked to both the key and subject specific competences. This is the ability to plan further steps in order to succeed solving a problem as well as dividing the process of solving more complex problems into smaller steps. A goal of teaching mathematics is to develop pupil's intuitions in mathematics appropriate for their age. The ability to understand and use mathematical concepts (e.g. angle, length, area, formulae and equations) is much more important than memorising formal definitions.

This syllabus has also been written so that it can be accessible by teachers, parents and pupils. This is one reason why icons have been used (listed in section 4.2.). These icons represent different areas of mathematics and are not necessarily connected to just one competency but can cover a number of competences.

To ensure pupils have a good understanding of the mathematics the courses from S1 to S7 have been developed linearly with each year the work from the previous year is used as a foundation to build onto. Thus, it is essential before commencing a year the preceding course must have been covered or a course that is similar. The teacher is in the best position to understand the specific needs of the class and before beginning a particular topic it is expected that pupils have the pre-required knowledge. A refresh is always a good idea when meeting a concept for the first time in a while. It should be noted that revision is not included in the syllabus, however, as mentioned earlier about limiting new content, there is time to do this when needed.

The use of technology and digital tools plays an important role in both theoretical and applied mathematics, which is reflected in this syllabus. The pupils should get the opportunity to work and solve problems with different tools such as spreadsheets, computer algebra system (CAS) software, dynamic geometric software (DGS), programming software or other software that are available in the respective schools. Technology and digital tools should be used to support and promote pupils' understanding, for example by visualising difficult concepts and providing interactive and personalised learning opportunities, rather than as a substitute for understanding. Their use will also lead to improved digital competence.

Teachers have full discretion with how to teach this course, materials to use and even the sequence the content is taught in. The content and the competencies (indicated in the tables in section 4.2., columns 2 and 3) to be covered is, however, mandatory.

The S6 Advanced course (3 Period)

This course has been specifically written for those who are choosing to study mathematics at a higher level, addressing topics and concepts that require higher level mathematics. It is also a course for pupils that love mathematics and enjoy going deeper into the theory.

The S6 Advanced course focusses on the foundations of Mathematics as well as on the basics of linear algebra. It also aims at deepening the knowledge of analysis and providing a wide picture of arithmetic. The topics chosen give the teachers the opportunity to engage their pupils in various activities, so that they continue to approach Mathematics as a tool for solving problems in a creative way and enjoy it.

Arithmetic will be entirely covered in S6, whereas algebra and analysis has been split between S6 and S7, thus giving more time to the teachers for recursive teaching and more opportunities to grasp the notions to the pupils.

The Advanced course is not only about applying theorems, it is mainly about understanding them. The teachers will therefore dedicate a significant amount of time to proving the critical theorems and results from the course.

The topic about the classical mathematical methods of demonstration must be applied as far as possible to all the content of this syllabus. This means that if some proofs are compulsory and must be known by the pupils, many other results for which the verb "prove" is not used in the learning objectives should nevertheless be proved in class, so that this syllabus is not only a collection of mathematical results, but rather a consistent journey into what Mathematics really are. For example, the classical theorems in the Analysis section are not disjointed theorems: following through the proof, jumping from one theorem to the following one, provides an overall understanding of the concepts and is far more memorable and enlightening for the pupils.

The S7 Advanced course (3 Period)

The study of Algebra and Analysis is strengthened in S7, the course being a continuation for both topics from the S6 Advanced course. These compulsory topics are completed with two optional ones chosen from a wide list. The goal of introducing many options is to give the pupils the opportunity to manipulate original concepts while getting ready for their higher studies. The teachers can adapt the choice of options to future needs of their pupils as closely as possible, depending on the higher education studies they will choose. The choice of options should therefore be done after a careful review of the applications that the pupils will have submitted to the universities and higher educational institutions.

About the necessity to prove as many theoretical results as possible in the classroom already mentioned for the S6 Advanced Course, it remains valid in S7 and pursues the same goal.

3. Learning Objectives
3.1. Competences

The following are the list of subject specific competences for mathematics. Here the key vocabulary is listed so that when it comes to reading the tables in section 4.2. the competency being assessed can be quickly seen. Please note that the list of key vocabulary is not exhaustive, and the same word can apply to more than one competency depending on the context. Further information about assessing the level of competences can be found in section 5.1. Attainment Descriptors.

The key concepts here are those needed to attain a sufficient mark.

	Competency	Key concepts (attain 5.0-5.9)	Key vocabulary
1.	**Knowledge and comprehension**	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Apply, classify, compare, convert, define, determine, expand, factorise, identify, know, manipulate, name, order, prove, recall, recognise, round, simplify, understand, verify
2.	**Methods**	Carries out mathematical processes in straightforward contexts, but with some errors	Apply, calculate, construct, convert, draw, manipulate model, plot, simplify sketch solve, use, verify
3.	**Problem solving**	Translates routine problems into mathematical symbols and attempts to reason to a result	Classify, compare, create, develop, display, estimate, generate, interpret, investigate, measure, model, represent, round, simplify, solve
4.	**Interpretation**	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Calculate, conduct, create, develop, discover, display, generate, interpret, investigate, model
5.	**Communication**	Generally presents reasoning and results adequately using some mathematical terminology and notation	Calculate, conduct, create, discover, display, interpret, investigate, model, present
6.	**Digital competence**	Uses technology satisfactorily in straightforward situations	Calculate, construct, create, display, draw, model, plot, present, solve

3.2. Cross-cutting concepts

Cross cutting concepts will be carried by the joint competences. The list of cross cutting concepts that will be composed will be shared by all science and mathematics syllabuses. The tentative list to be taught is based on the next generation science standards in the United states (National Research Council, 2013):

	Concept	Description
1.	**Patterns**	Observed patterns of forms and events guide organisation and classification, and they prompt questions about relationships and the factors that influence them.
2.	**Cause and effect**	Mechanism and explanation. Events have causes, sometimes simple, sometimes multifaceted. A major activity of science is investigating and explaining causal relationships and the mechanisms by which they are mediated. Such mechanisms can then be tested across given contexts and used to predict and explain events in new contexts.
3.	**Scale, proportion and quantity**	In considering phenomena, it is critical to recognise what is relevant at different measures of size, time, and energy and to recognise how changes in scale, proportion, or quantity affect a system's structure or performance.
4.	**Systems and system models**	Defining the system under study—specifying its boundaries and making explicit a model of that system—provides tools for understanding the world. Often, systems can be divided into subsystems and systems can be combined into larger systems depending on the question of interest
5.	**Flows, cycles and conservation**	Tracking fluxes of energy and matter into, out of, and within systems helps one understand the systems' possibilities and limitations.
6.	**Structure and function**	The way in which an object or living thing is shaped and its substructure determine many of its properties and functions and vice versa.
7.	**Stability and change**	For natural and built systems alike, conditions of stability and determinants of rates of change or evolution of a system are critical for its behaviour and therefore worth studying.
8.	**Nature of Science**	All science relies on a number of basic concepts, like the necessity of empirical proof and the process of peer review.
9.	**Value thinking**	Values thinking involves concepts of justice, equity, social–ecological integrity and ethics within the application of scientific knowledge.

In the mathematics syllabuses, the concepts 5 and 8 will be addressed only to a limited extent.

The lists of competences and cross cutting concepts will serve as a main cross-curricular binding mechanism. The subtopics within the individual syllabuses will refer to these two aspects by linking to them in the learning goals.

4. Content

4.1. Topics

This section contains the tables with the learning objectives and the mandatory content for the strand Mathematics Advanced in S6 and S7 (3 periods per week).

4.2. Tables

How to read the tables on the following pages

The learning objectives are the curriculum goals. They are described in the third column. These include the key vocabulary, highlighted in bold, that are linked to the specific mathematics competences found in section 3.1. of this document.

These goals are related to content and to competences. The mandatory content is described in the second column. The final column is used for suggested activities, key contexts and phenomena. The teacher is free to use these suggestions or use their own providing that the learning objective and competencies have been met.

Please note that the word ‘limitation’ is used to ensure that when extension is planned it is planned with the idea of horizontal extension rather than vertical extension as mentioned in section 2. of this document.

Use of icons

Furthermore, there are six different icons which indicate the areas met in the final column:

	Activity
	Cross-cutting concepts
	Digital competence
	Extension
	History
	Phenomenon

Each of these icons highlight a different area and are used to make the syllabus easier to read. These areas are based on the key competences mentioned in section 1 of this document.

YEAR 6 (AM)	**TOPIC: FOUNDATIONS OF MATHEMATICS**		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Set theory	Relations between sets	**Know** and **use**: • the concept of sets and subsets • equality of two sets • relationship between the union and the intersection of sets (distributive of one over the other and De Morgan's laws) • set of all possible subsets • cartesian product of sets *Note: remind the pupils the concepts of universal set, empty (null) set, complement of a set, union and intersection of sets*	Prove the relationship between the union and the intersection of sets using Venn diagrams
		Determine the cardinal number of a set, including the set of all possible subsets for a finite set	Compare the cardinal numbers of $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$ and an interval of $\mathbb{R}$, e.g. $\left]-\frac{\pi}{2};\frac{\pi}{2}\right[$, and discover the idea of countable set
Vocabulary, reasoning and proof	Basics of mathematical logic	**Distinguish** propositions (statement) and predicates **Recognise** whether a proposition is true or false **Determine** the truth table of: • the conjunction or the disjunction of two propositions • an implication and an equivalence • the negation of a proposition and of an implication	Explore what constitutes an axiom, a lemma, a theorem and a corollary

YEAR 6 (AM)	TOPIC: FOUNDATIONS OF MATHEMATICS			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Understand that the negation of an implication is not an implication **Recognise** a necessary and/or sufficient condition **Know** and **use** the existential and the universal quantifiers **Determine** the negation of a proposition which includes existential and/or universal quantifiers		
	Classical mathematical methods of demonstration	**Implement** the classical mathematical methods of demonstration, especially to demonstrate an equality: • *"modus ponens"* and "*modus tollens"* inference rules • proof by contraposition • deduction theorem • proof by separation of cases • proof by contradiction ("*reductio ad absurdum"*) • proof by induction **Use** appropriately a counter example	 	• The axiomatic method and Euclid's fifth postulate • Descartes axiomatic approach to Philosophy • The American Declaration of Independence "we hold these truths to be self-evident…" • Russell's paradox (use for example the metaphor of the Catalogue Paradox) • Brouwer, constructivism and the law of the excluded middle: $\hat{\pi}$ ("pi hat") Discussion of nonconstructive proofs

YEAR 6 (AM)	**TOPIC: FOUNDATIONS OF MATHEMATICS**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Relations and functions	Binary relations	**Know** and **apply** the concept of binary, order and equivalence relations **Recognise** order and equivalence relations		
	Functions	**Recognise**: • codomain, image and preimage by a function • injective, surjective, bijective and inverse functions		Explore how the concepts of range, image and codomain relate to each other
		Understand and **apply** the pigeonhole (Dirichlet's) principle		$p_1, p_2, \ldots, p_n$, are the first n positive integers arranged at random. If n is odd, prove that $(1-p_1)\cdot(2-p_2)\cdot \ldots \cdot(n-p_n)$ is even

YEAR 6 (AM)	TOPIC: ARITHMETIC		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Euclidean division and congruences	Euclidean division	**Know** the Euclidean division (existence and uniqueness of the quotient and the remainder)	• Develop the algorithm for finding the number of factors of a composite positive integer and write a program to implement it • Write a (recursive) program which returns the quotient and the remainder of a Euclidean division using only addition and subtraction
	Congruences	**Understand** the meaning of congruence modulo n, i.e. $a \equiv b[n]$ with $a, b, n \in \mathbb{Z}$ $(n \geq 2)$ **Prove** and **know** that congruence modulo n is an equivalence relation on the set of integers	
		Prove and **apply** the following congruence theorems: for $a, b, c, d, n \in \mathbb{Z}$ $(n \geq 2)$and $p \in \mathbb{N}$ (including 0), if $a \equiv b[n]$ and $c \equiv d[n]$, then: • $a \pm c \equiv b \pm c[n]$ • $a \pm c \equiv b \pm d\ [n]$ • $a \cdot c \equiv b \cdot d\ [n]$ • $a^p \equiv b^p\ [n]$	• Use congruence theorems to demonstrate various well-known divisibility criteria (2, 3, 5 and 9) • Use congruence theorems to find the remainder of a Euclidean division, e.g. 7^{2021} divided by 15 Explore various non common divisibility criteria (7, 11, 13, 17, 19, …)
Prime numbers	Prime numbers	**Prove** and **know** that the set of prime numbers is infinite	• Explore the distribution of prime numbers, e.g. Ulam spiral • Explore twin primes numbers
	Prime factor decomposition	**Know** the theorem about the existence and the uniqueness of the prime factor power decomposition of an integer	

YEAR 6 (AM)	TOPIC: ARITHMETIC		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
		Determine with a technological tool the prime factor power decomposition of an integer and in particular whether an integer is prime or not	
	Greatest common divisor and least common multiple	**Prove** and **apply** the Euclidean algorithm to determine the greatest common divisor (gcd) of two integers	Write a program corresponding to the Euclidean algorithm
		Determine whether two integers are coprime or not	
		Determine the greatest common divisor and the least common multiple (lcm) of two or more integers using the prime factor power decomposition	Prove the formula $\text{gcd}(a,b)\cdot\text{lcm}(a,b)=\lvert a\cdot b\rvert$ for $a,b\in\mathbb{Z}$
	Bézout's identity	**Apply** Bézout's identity for two integers a and b: for $d=\text{gcd}(a,b)$, there exist integers x and y such that $a\cdot x+b\cdot y=d$	
	Bachet-Bézout's theorem	**Apply** Bachet-Bézout's theorem: two numbers a and b are coprime if and only if there exist integer numbers x and y such that $a\cdot x+b\cdot y=1$	
	Gauss' lemma	**Apply** Gauss' lemma: for $a,b,c\in\mathbb{Z}$, if a and b are coprime and if a divides $b\cdot c$, then a divides c	Apply Bachet-Bézout's theorem and the lemma: • to solve a linear Diophantine equation • to prove the Chinese remainder theorem
	Euclid's theorem	**Apply** Euclid's theorem: if a prime number p divides a product $a\cdot b$, then p divides a or p divides b	

YEAR 6 (AM)	TOPIC: ARITHMETIC		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
	Fermat’s little theorem	**Apply** Fermat’s little theorem: if p is a prime number coprime with an integer a, then $a^{p-1} \equiv 1[p]$	Use a spreadsheet to develop, state and prove the Euler-Fermat theorem and show Fermat’s little theorem as a special case. Explore • various encryption methods, in particular RSA • the concept of probable prime numbers History of cryptography

YEAR 6 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Vector Spaces	Definition of a vector space	**Define** a vector space and distinguish between addition of vectors and multiplication by a scalar *Notes:* • *The pupils do not know what a group/field is. The structure of a group should be briefly described.* • *The definition of the algebraic structure of the field will be briefly described and illustrated with simple examples familiar to pupils ($\mathbb{Q}$, $\mathbb{R}$ or $\mathbb{C}$ and usual operations)* **Know** the following properties: • $(-1)\vec{u} = -\vec{u}$ • $\lambda\vec{u} = \vec{0} \Leftrightarrow \lambda = 0$ or $\vec{u} = \vec{0}$		
	Examples of vector spaces	**Know** some examples from the past years: • vector space of polynomials • continuous functions on $\mathbb{R}$ • one-dimensional vector space: a straight line • 2-dimensional vector space: a plane • 3-dimensional vector space: all polynomials of degree less than or equal to 2 • 4-dimensional vector space: all polynomials of degree less than or equal to 3 • $\mathbb{R}^n$ and $\mathbb{C}^n$	 	• The number sets: which one of these is a vector space on another one, e.g. is $\mathbb{R}$ a $\mathbb{C}$-vector space? • Explore projections of 3-dimensional spaces onto 2-dimensional spaces Use a technological tool to visualise the effect of the addition and the scalar multiplication on vectors in $\mathbb{R}^3$

YEAR 6 (AM)	TOPIC: LINEAR ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
		Calculate linear combinations of vectors	
	Vector Subspaces	**Define** a subspace	Explore subspaces of polynomials of degree smaller than or equal to a given natural number
		Specify sufficient conditions for a set to be a subspace	
Dimension of a vector space	Generating set and basis	**Investigate** generating independent and dependent sets for a vector space	
		Explore the relationships between generating sets and independent sets	Show that $\mathbb{C}$ is a two-dimensional real vector space
		Recognise a basis	Investigate the set of functions $f_a(x) = \|x - a\|$ as vectors in the real vector space of continuous functions
		Define the dimension of a vector space	
	Rank of a set of vectors	**Determine** the rank of a set of vectors	Use some of the previous examples of sets of vectors and calculate their rank
Basic concepts of matrices	$m \times n$ matrix with elements $(a_{i,j})$	**Define** a $m \times n$ matrix with elements $(a_{i,j})$ including square matrices, triangular matrices, diagonal matrices	
		Know the rules about adding two $m \times n$ matrices and multiplying an $m \times n$ matrix by a scalar	
	The set of matrices as a vector space	**Prove** that the set of $m \times n$ matrices is a vector space	

YEAR 6 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Find a basis for the set of $m \times n$ matrices		
		Investigate the existence of basis with triangular or diagonal matrices for 2×2 or 3×3 square matrices		
	Rank of a matrix	**Define** the rank of a matrix		
		Calculate the rank of a matrix for some examples with $m \leq 3$ and $n \leq 3$		Use a technological tool to calculate the rank of a $m \times n$ matrix for some examples with $m > 3$ or $n > 3$
	Transposed matrix	**Illustrate** with some examples that the rank of a matrix is equal to the rank of its transposed matrix		
Operations on matrices	Product of matrices	**Know** the rule for multiplying matrices		Describe, using matrices and for a small number of pages, the Pagerank algorithm
		Discuss the commutativity and associativity of the product of matrices using a technological tool if necessary		• Find for a given matrix A, all matrices that commute with A, i.e.: $AX = XA$ • Prove that if A and B are two square matrices such that $AB = A + B$, then A and B commute
	Block matrices	**Discuss** conditions on the dimensions of the submatrices so that if $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ and $N = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix}$, then $M \times N = \begin{pmatrix} AA' + BC' & AB' + BD' \\ CA' + DC' & CB' + DD' \end{pmatrix}$		
	Square matrices	**Interpret** some usual systems of equations using square matrices		Transform the problem of finding the equation of a parabola into a matrix problem
		Calculate powers of square matrices		

YEAR 6 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Inverse of a square matrix	**Define** the inverse of a square matrix		Write a Fibonacci sequence as a matrix problem and deduce the general term of the sequence
		Determine the inverse of a square matrix for some specific matrices		• Derive the inverse of a block matrix $M = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$ from the product of block matrices • Deduce the inverse of $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ by squaring it • Deduce the inverse of $B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ by squaring it • Apply the concept of invertible matrices and modular arithmetic to the Hill cipher

YEAR 6 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Systems of equations: Gaussian elimination	Equivalent systems of equations	**Study** systems by Gaussian elimination method (exchange of two rows, multiplication of a row by a nonzero number, adding a multiple of a row to another row)		
	Compatible, incompatible, determinate and indeterminate systems	**Transform** a system of equations into an equivalent triangular form		
		Form, from a concrete problem, a system of equations, solve this system and interpret the solution obtained		
		Use a technological tool to solve systems with more than 3 equations and 3 unknowns		
		Interpret the operations in a Gaussian elimination process in terms of matrices		Practise multiplying by permutation matrices (left and right)

YEAR 6 (AM)	TOPIC: ANALYSIS			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Classical theorems	Continuity and differentiability	**Understand** intuitively the concept of continuity. **Investigate** functions that are not continuous and/or not differentiable at a specific point.		For example, the absolute value, cube root, and floor functions
	Derivative of an inverse function	**Know** and **prove** the formula about the derivative of an inverse function, i.e. $(f^{-1})' = \frac{1}{f' \circ f^{-1}}$ **Derive** the formula of the derivative for the inverse trigonometric functions		
	Intermediate value theorem	**Apply** the intermediate value theorem: consider a continuous function f on an interval $I = [a, b]$; if u is a number between $f(a)$ and $f(b)$, then there exists at least one $c \in (a, b)$ such that $f(c) = u$		
	Rolle's theorem	**Apply** and **interpret** Rolle's theorem: if a function f is continuous a closed interval $[a; b]$ and differentiable on the open interval (a, b) with $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$		
	Mean value theorem	**Apply** and **interpret** geometrically the mean value theorem: if a function f is continuous on the closed interval $[a; b]$ and differentiable on the open interval (a, b), then there exists at least one $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$		A four-legged table on an uneven floor can be moved around to a position where it does not wobble

YEAR 6 (AM)	TOPIC: ANALYSIS			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Apply and **interpret** geometrically the mean value inequality and its corollaries: consider a function f, continuous on the closed interval $[a;b]$ and differentiable on the open interval (a,b): • if there exist two values m and M such that for all $x \in (a,b)$, $m \leq f'(x) \leq M$, then $m \leq \dfrac{f(b)-f(a)}{b-a} \leq M$ • if there exists a value M such that for all $x \in (a,b)$, $\lvert f'(x)\rvert \leq M$, then $\left\lvert\frac{f(b)-f(a)}{b-a}\right\rvert \leq M$ • if there exists a value M such that for all $x \in (a,b)$, $\lvert f'(x)\rvert \leq M$, then for all $x, y \in (a,b)$ with $x \neq y$, $\lvert f(y)-f(x)\rvert \leq M\lvert y-x\rvert$		
	Lipschitz function	**Define** a Lipschitz function **Determine** a fixed point of a differentiable Lipschitz function **Represent** a sequence tending to a fixed point of a differentiable Lipschitz function		Write a program to determine an approximate value, with a given number of digits, of a fixed point of a differentiable Lipschitz function

YEAR 7 (AM)	**TOPIC: LINEAR ALGEBRA**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Finite-dimensional vector spaces and linear transformations	Linear transformations and vector spaces	**Define** a linear transformation		Prove that the set of linear transformations between two vector spaces is a vector space
		Investigate the image and the preimage of a subspace by a linear transformation		
		Recognise two specific subspaces: the kernel and the image of a linear transformation		
	Isomorphism	**Determine** conditions on the kernel and on the image for the linear transformation to be injective, surjective or bijective		Describe the isomorphism between a n-dimension vector space and $\mathbb{R}^n$ or $\mathbb{C}^n$
	Linear transformations and matrices	**Construct** the matrix of a linear transformation $f : E \to F$ between vector spaces in finite dimension given a basis of E and a basis of F		Deduce the dimension of $\mathcal{L}(E, F)$ from the dimension of $M_{n,p}(K)$
		Investigate the effect of a change of basis for E and for F: • on the elements of the matrix • on the rank of the matrix		Explore differentiation as a linear transformation between subspaces of polynomials of dimension smaller than or equal to n
		Interpret the sum and product of matrices as operations on linear transformations		

YEAR 7 (AM)	TOPIC: LINEAR ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Determinants	Determinants and matrices	**Calculate** the determinants of 2×2 and 3×3 square matrices by using the products of diagonals (Sarrus rule) **Understand** that this is an algorithm that results from the solution of linear systems of 2 equations in 2 variables and 3 equations in 3 variables	
	Cofactors, minors and submatrices	**Calculate** the determinant of a 3×3 matrix using cofactors, minors and submatrices	**Calculate** a determinant of order $n \geq 4$ with a technological tool
		Know that the determinant of a square matrix is equal to the sum of the products obtained by multiplying the elements of any row (column) by their respective cofactors	Deduce some cases where the determinant is equal to zero: • null row / null column • at least two identical columns / rows • a row (or column) is a multiple of another one
		Apply the following properties to the calculation of a determinant of order n (for simplicity the following apply to order 3 with columns $C1$, $C2$, $C3$): • $\det(C1, C2, C3) = -\det(C2, C1, C3)$ • $\det(rC1, C2, C3) = r\det(C1, C2, C3)$ • $\det(C'1 + C''1, C2, C3) = \det(C'1, C2, C3) + \det(C''1, C2, C3)$ • $\det(C1 + rC2 + sC3, C2, C3) = \det(C1, C2, C3)$ $\det(M \times N) = \det(M) \times \det(N)$ for two $n \times n$ square matrices	Calculate the determinant of a Vandermonde matrix

YEAR 7 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Application to the rank of a matrix	**Determine** the rank of a matrix by using the determinants of submatrices **Use** determinants to verify the linear dependence or independence of matrix rows or columns		
Inverse of a non-singular square matrix		**Check** whether a given matrix is non-singular or singular **Determine** the inverse of a square matrix using the determinant, the co-factor matrix and the adjugate matrix **Solve** systems of linear equations by writing them in matrix form and applying the inverse matrix		Justify that all non-singular square matrices of order 2 form a noncommutative multiplicative group
Systems of equations		**Know** the general theorem to solve systems of $m \times n$ linear equations (Rouché-Fröbenius theorem) **Apply** Cramer's rule to solve systems of $m \times n$ linear equations **Apply** Rouché-Fröbenius theorem and Cramer's rule to solve systems of linear equations of one or more parameters		Solve $n \times n$ systems of linear equations by using Cramer's rule

YEAR 7 (AM)	TOPIC: LINEAR ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Solve homogeneous systems of equations *Limitation: all exercises without a technological tool will be limited to cases of simple systems of* 2 *or* 3 *equations with to* 2 *or* 3 *variables*		

YEAR 7 (AM)	TOPIC: ANALYSIS			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Taylor and Maclaurin formulae	n^{th} derivative	**Calculate** the n^{th} derivative of a n-times differentiable function		
	Expansion of order *n* of a function	**Understand** the concept of an expansion of order *n* of a function around a value.	 	Explore: - expansion of order n of polynomials - expansion of orders 1 and 2 and interpret them graphically Explore with a technological tool the increasing accuracy of the expansion of the sine function and/or that of the cosine function as the order of the expansion increases and point out a connection in terms of the parity of the functions.
	Taylor and Maclaurin polynomials	**Apply** the Taylor and Maclaurin polynomials of order n with Lagrange remainder to a $(n+1)$-times differentiable function to give an approximate value of the function and the corresponding error bounds **Determine** for the following functions their Maclaurin polynomials of order n with the Peano form of the remainder: • $x \mapsto \frac{1}{1\pm x}$ • $x \mapsto \ln(1 \pm x)$ • $x \mapsto (1 \pm x)^n,\ n \in \left\{\frac{1}{2}, 2,3,4, \dots\right\}$ • $x \mapsto \mathrm{e}^x$ • $x \mapsto \cos(x)$ • $x \mapsto \sin(x)$		

YEAR 7 (AM)	**TOPIC: ANALYSIS**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Determine Maclaurin polynomials of order n of functions obtained by sum, difference, product and/or composition of the functions listed above *Limitation: determine Maclaurin polynomials of order n of functions obtained by division or integration is not compulsory*		• Apply expansions to: ○ find an approximation of a function around a point with a polynomial of degree equal to or greater than 2 ○ determine limits including the asymptotic behaviour of a function ○ determine the relative position of the graph of a function with respect to one of its tangents
Integration techniques	Inverse functions	**Calculate** integrals using the Arccosine, Arcsine and Arctangent functions		
	Division of polynomials	**Calculate** integrals of the type $\int \frac{P(x)}{Q(x)} dx$ where P and Q are polynomial functions including improper integrals *Limitation: study only cases in which the polynomial Q* • *has one or several single roots* • *has one unique multiple root* • *is of the type $a \cdot x^2 + b \cdot x + c$* with $\Delta = b^2 - 4 \cdot a \cdot c < 0$		
	Use of recursive formulae	**Calculate** integrals using recursive formulae: for $m, n \in \mathbb{N}$ (including 0) For example: • $\int_0^{\frac{\pi}{2}} x^n \cdot \sin x \, dx$ • $\int_0^{+\infty} x^n \cdot \mathrm{e}^{-x} dx$		Calculate other integrals using recursive formulae, e.g. for $m, n \in \mathbb{N}$ (including 0): • $\int_0^{\frac{\pi}{2}} \cos^m x \cdot \sin^n x \, dx$ • $\int_1^{\mathrm{e}} (\ln x)^n dx$

YEAR 7 (AM)	TOPIC: ANALYSIS		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Differential equations	Differential equations	**Know** the concept of differential equations, homogeneous or not **Know** that the general solution of a non-homogeneous linear differential equation is formed by the sum of a particular solution with the general solution of the associated homogeneous equation	Explore whether the solutions of a (non) homogeneous differential equation define a vector space or not
	First-order and second-order homogeneous linear differential equations	**Solve** a first order and a second order homogeneous linear differential equation with constant coefficients including cases with respect to initial conditions	Solve equations linked to real–life situations, e.g.: • amount of money on a saving account which pays interest per annum • growing population with given rate grow • decaying of a radioactive material • temperature of a metal bar • velocity of a falling mass Explore first-order and/or second-order linear differential equations with constant coefficients including cases with respect to initial conditions *Note: in this case, the form of the particular solution must be given* Investigate the process of solving differential equations using a technological tool

S7 – Advanced Mathematics (3P) – OPTIONAL TOPICS

IMPORTANT NOTE:

Unlike the preceding compulsory part of the program the description of each optional topic gives only a general overview of the content. Small adjustments in the content, linked to specific programs or requirements of national universities in different countries of the European Union remain possible. It is up to the teacher to make the necessary changes.

However, for the sake of readability and comparability of this part of the program, teachers in charge of this course must keep an accurate record of the adjustments made to the chosen options. This record will accompany the oral exam questions forwarded to the inspector responsible for mathematics in the European Schools. This will ensure that all such information (statement of the subject matter and the oral exam) is available to external examiners appointed for the oral tests.

1. Algebraic structures

- internal composition law, associativity, identity element, invertible element, inverse element
- group, abelian group, subgroup including its characterization
- ring, subring including its characterization
- field
- various examples of finite or infinite groups, subgroups, rings, subrings and fields (including $\mathbb{Z}/n\mathbb{Z}$)
- groups or ring homomorphisms, endomorphisms, isomorphisms and automorphisms

2. Algorithms and programming

- basic algorithms
- local and general variables
- definition of function and program
- management of inputs and outputs
- control statements:
  - conditional statements ("if ... then ..." "repeat ... until ... " "do … while ... ")
  - instructions in loops ("for ... ranging from ... to ... ")
- compiling a program
- applications in practical situations (analysis, numerical analysis, probability, statistics, geometry, ...)

3. Applications of integration

- calculation of the root-mean-square value of a function
- finding the position of the centroid of a plane area, symmetric solid or shell
- calculation of moment of inertia of a plane area, symmetric solid or shell
- length of an arc and area of a surface of revolution
- polar integration

4. Artificial intelligence and machine learning

- classification and regression problems
- decision trees
- Naive Bayes classifier, examples: spam-filtering and sentiment analysis
- k-nearest neighbour algorithm for classification and regression
- the artificial neuron: input, output, weights, bias and different activation functions
- artificial neural networks with at least three layers
- artificial neural networks as universal function approximators
- different loss functions for artificial neural networks
- gradient descent for updating weights
- problems: overfitting, curse of dimensionality

5. Barycentric calculus and affine (linear) geometry

- barycentric calculus
- study of the functions $M \mapsto \sum_{i=1}^{n} \alpha_i \overrightarrow{MA_i}$ and $M \mapsto \sum_{i=1}^{n} \alpha_i \cdot \left\|\overrightarrow{MA_i}\right\|^2$
- affine functions
- particular cases:
    - isometrics
    - translations
    - enlargements
    - symmetries
    - orthogonal or not affinities

6. Classical geometry

- study of configurations, parallelism, orthogonality
- construction problems
- minimal path problems
- use of transformations
- matrices of transformations
- changing axes translation, rotation of orthonormal axes

7. Confidence Intervals, hypothesis tests and chi-squared test

- unbiased estimates of the population mean, μ, and variance, σ^2
- distribution of the sample mean $\overline{X}$, when X is normally and non-normally distributed (central limit theorem)
- confidence intervals for the mean, μ, of a normal population with known variance and unknown variance (large sample); of a non-normal distribution with known and unknown variance (large sample)
- hypothesis testing for the parameter, p, of a binomial distribution (small sample) and for the mean, λ, of a Poisson distribution
- hypothesis testing for the mean μ, of a normal population with known variance and unknown variance (large sample); of a non-normal distribution with known and unknown variance (large sample)

- setting up a null hypothesis and choosing a one- or two-tailed alternative hypothesis, stating a critical value (significance level)
- rejecting or accepting the null hypothesis based on the test outcome
- calculating the type 1 and type 2 errors
- the χ^2 significance test for independence and χ^2 goodness-of-fit test for the binomial, Poisson and Normal distributions

8. Conics

- definition by directrix and eccentricity
- Cartesian equation and reduced equation
- parabola, circle, ellipse, hyperbola and degenerate conics
- establishment of a conic from an equation of the form

$$A \cdot x^2 + B \cdot x \cdot y + C \cdot y^2 + 2 \cdot D \cdot x + 2 \cdot E \cdot y + F = 0$$

9. Descriptive geometry

- point, line, plane
- changing the plane of projection
- intersections of lines and planes
- orthogonality of lines and planes
- geometrical problems in space

10. Dynamics of a point in the plane

- system of reference, movement of a point
- trajectory, velocity vector, acceleration vector
- composition of velocities, of accelerations

11. Graph theory

- definition
- undirected and directed graphs
- weighted graphs
- adjacent matrices
- Eulerian graphs
- Hamiltonian graphs
- spanning trees
- Dijkstra algorithm
- Markov chains and graphs

12. Isometries of the affine 3D-Euclidean space

- link between vector isometries and affine isometries
- study and classification of isometries

13. Linear and multilinear forms

- dual space of a vector space
- covectors
- dual basis
- linear forms and change of basis
- vector lines and planes in 3-dimensional space
- multilinear forms
- symmetric multilinear forms, alternate forms: determinants in 2-dimensional and 3-dimensional space
- determinant of an endomorphism
- effect of a change of basis
- linear independence

14. Mechanics

- acceleration
- Newton’s laws including Newton’s law of restitution
- vectors
- moments
- impulse and momentum
- coefficient of friction
- projectiles
- work, energy and power
- centre of mass
- toppling
- elastic strings
- simple harmonic motion
- motion in a circle
- motion in a vertical circle

15. Non-linear systems

- sensitivity to initial conditions
- Feigenbaum-curve and the Mandelbrot set
- attractors
- Newton-iterations

16. Parametric equations and polar coordinates

- parametric equations and polar coordinates: definition, domain, range, symmetry, asymptotes, derivative, tangents, second derivative, direction in which the plane curve is traced as the parameter increases, graph a curve, area, arc length and surface area
- transform a parametric equation to a cartesian equation and vice-versa
- transform a polar equation to rectangular coordinates
- line, circle, ellipse, cycloids and other parametric curves
- circle, cardioid, cissoid, conchoid, lemniscate, limaçon, logarithmic spiral, rose and other polar curves

17. Partial differentiation

- functions of two variables
- first order partial derivatives
- geometrical interpretations
- higher order derivatives
- Euler's first theorem for homogeneous functions
- differentials
- differentiation of composite functions
- directional derivatives - maxima, minima, saddle points

18. Plane Intersections of surfaces

- functions of two variables
- intersection of a surface by a plane
- intersection of a cylinders
- intersection of a cones
- the equation of an intersecting surface $z = x^2 + y^2$ and $z = x \cdot y$

19. Polynomials

- vector space and ring of polynomials
- division using decreasing powers: uniqueness, quotient and remainder
- H.C.F. of two polynomials
- zeros of polynomials
- polynomials in several variables

20. Representation of numbers and binary arithmetic

- binary representation of positive integers including binary addition and binary multiplication
- hexadecimal, octal, bits, bytes and words including conversions
- representation of positive and negative numbers
  - sign-magnitude method
  - signed binary integers (2's complement)
  - positive binary fractions
  - signed binary fractions
  - fixed point number representation (2's complement)
  - floating point number representation (2's complement)
  - range of floating-point representation
  - normalisation of floating-point numbers
  - rounding modes
- floating point arithmetic operations including addition and subtraction, multiplication and division
- accuracy problems including machine precision and minimizing the effect of accuracy problems

21. Series

- definition, general term of a series, *n*-th partial sum (limited to series of real numbers)
- upper and lower bounds of a series
- convergence and divergence of a series of positive terms, series of negative terms, alternating series,
- a necessary condition of convergence of a series: if the series $\sum u_n$ converges, then the sequence (u_n) converges to zero, the converse is false
- some classic number series:
  - the geometric series $\sum a^n$ with $a \in \mathbb{R}$
  - the harmonic series $\sum \frac{1}{n}$
  - the alternating harmonic series $\sum \frac{(-1)^n}{n}$
  - the Riemann-series $\sum \frac{1}{n^\alpha}$ with $\boldsymbol{\alpha} \in \mathbb{R}$
- convergence criteria for numerical series:
  - convergence criteria for geometric series
  - comparison criteria for series of positive terms
  - convergence criterion for alternate series (Leibniz rule): if (u_n) is an alternating sequence such that $(|u_n|)$ a decreasing sequence and $\lim_{n\to\infty} u_n = 0$, then the series $\sum u_n$ converges
  - convergence criterion for Riemann-series: if $\alpha > 1$ then a series with the general term $\frac{1}{n^\alpha}$ converges and if $\alpha \leq 1$ it diverges
  - d'Alembert's converge criteria: if $\lim_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = 1$ then the series with a general term (u_n) converges if $l < 1$, diverges if $l > 1$ and is indeterminable if $l = 1$
- absolutely convergent series and semi-convergent series

22. Similarities and isometries

- geometrical interpretation of operations on complex numbers
- functions $f(z) = a \cdot z + b$ and $g(z) = a \cdot \bar{z} + b$
- direct and indirect similarities
- direct and indirect isometries

23. Simplex algorithm

- formulation of a linear programming problem: objective function, constraints
- artificial variables: inflows or outflows variables
- matrix formulation of a problem
- background variables and non-basic variables
- solutions eligible basic primal solution
- simplex algorithm: pivot method
- optimal solution
- special cases
- method of two phases

24. Special relativity (in two dimensions)

- kinematic diagrams - time invariance: Galilean group of matrices $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- velocity addition $w = u + v$
- isomorphisms of (G, x) and of $(R, +)$
- natural units $(c = 1)$
- velocity of light invariance: Lorentz group of matrices $\beta \begin{pmatrix} 1 & u \\ u & 1 \end{pmatrix}$ with $\beta = \frac{1}{\sqrt{1-u^2}}$
- velocity addition $w = \frac{u+v}{1+u \cdot v}$
- conditions $\|\vec{u}\| < 1$ and $\|\vec{w}\| > 1$
- tachyons - contraction, dilatation, Doppler effects

25. Topological ideas

- intuitive topology (points, arcs, surfaces)
- from intuitive topology to structured topology
- topological spaces (models: topology of discs, of parallelepipeds, of spheres)
- Hausdorff space
- homeomorphisms

26. Trigonometric and hyperbolic functions

- inverse trigonometric functions (arcsine, arccosine and arctangent) and hyperbolic functions (hyperbolic sine, hyperbolic cosine and hyperbolic tangent): definition, domain, limits, whether they are odd or even, domain for which they are differentiable, derivatives, where they are increasing/decreasing, graphs
- (families of) functions involving trigonometric or hyperbolic functions
- usual trigonometric and hyperbolic formulae and transformations:
  - for (hyperbolic) sine, (hyperbolic) cosine and (hyperbolic) tangent: square law formulae, sum/difference identities, double angle identities, product identities, sum to product identities, t-formula, … (all the trigonometric formulae can be recalled or derived using complex numbers)
  - linearize an expression involving trigonometric or hyperbolic functions
  - express:
    - $\cos(\mathrm{n}x)$ and $\sin(\mathrm{n}x)$ as a power of $\cos x$ and $\sin x$
    - $\cosh(\mathrm{n}x)$ and $\sinh(\mathrm{n}x)$ as a power of $\cosh x$ and $\sinh x$
- equations solved by using these formulae or transformations

27. Vector functions

- functions from $\mathbb{R}$ to $\mathbb{R}^2$ (or $\mathbb{C}$) or to $\mathbb{R}^3$
- differential coefficient of a vector function
- differentiation of the product of a real function by a vector function
- differentiation of a scalar product
- differentiation of a vector product
- construction of plane curves

28. Vector isometries in 3-dimensional space

- isometry and the associated matrix
- the orthogonal group: norm, scalar product and bases
- eigenvalues and eigenvectors
- composition of vector isometrics
- classification of vector isometrics

5. Assessment

For each level there are attainment descriptors written by the competences, which give an idea of the level that pupils have to reach. They also give an idea of the kind of assessments that can be done.

With the competences are verbs that give an idea of what kind of assessment can be used to assess that goal. In the table with learning objectives these verbs are used and put bold, so there is a direct link between the competences and the learning objectives.

Assessing content knowledge can be done by written questions where the pupil has to respond on. Partly that can be done by multiple choice but competences as constructing explanations and engaging in argument as well as the key competences as communication and mathematical competence need open questions or other ways of assessing.

An assignment where pupils have to use their factual knowledge to make an article or poster about a (broader) subject can be used to also judge the ability to critically analyse data and use concepts in unfamiliar situations and communicate logically and concisely about the subject.

In Europe pupils must have some competence in designing and/or engineering (STEM education). So there has to be an assessment that shows the ability in designing and communicating. A design assessment can also show the ability in teamwork.

Pupils have to be able to do an (experimental) inquiry. An (open) inquiry should be part of the assessments. Assessing designing and inquiry can be combined with other subjects or done by one subject, so pupils are not obliged to do too many designing or open inquiry just for assessment at the end of a year.

Digital competence can be assessed by working with spreadsheets, gathering information from internet, measuring data with measuring programs and hardware, modelling theory on the computer and comparing the outcomes of a model with measured data. Do combine this with other assessments where this competence is needed.

Assessment is formative when either formal or informal procedures are used to gather evidence of learning during the learning process and are used to adapt teaching to meet pupil needs. The process permits teachers and pupils to collect information about pupil progress and to suggest adjustments to the teacher's approach to instruction and the pupil's approach to learning.

Assessment is summative when it is used to evaluate pupil learning at the end of the instructional process or of a period of learning. The purpose is to summarise the pupils' achievements and to determine whether, and to what degree, the pupils have demonstrated understanding of that learning.

For all assessment, the marking scale of the European schools shall be used, as described in "Marking system of the European schools: Guidelines for use" (Ref.: 2017-05-D-29-en-7).

Examining the Advanced mathematics course

The S6 and S7 Advanced mathematics course is not only to be assessed using the oral baccalaureate exam and B tests but also through continuous teacher assessment using the various competences. These continuous, formative assessments can therefore vary in type and structure.

It is the responsibility of the teachers to clarify whether the use of a technological tool is allowed or not during B tests given in S6 and S7 according to the way the assessed topics have been taught. The information must be given to the pupils before the examination takes place.

Each oral examination question should clearly state whether the use of a technological tool is allowed or not. Partial use of a technological tool during an oral examination is not allowed. If the oral examination question does not allow the use of a technological tool then the candidate must hand in their technological tool to the teachers after the choice of question and, if the use is allowed, the teacher must check that it is in exam mode before the candidate goes to the preparation room.

Concomitantly to these obligations, teachers in charge of this course will write the oral questions with respect to the following guidelines:

- The proposals correspond to the mathematics syllabuses in year 6 and 7.
- The questions appropriately reflect the competences described in the mathematics syllabuses in year 6 and 7.
- The questions have a good balance of elements and examine the general understanding of the evaluated topics.
- Presenting answers to all questions takes for every student maximum 20 minutes.

Note that cascading questions are allowed: if pupils have some difficulty to answer a question, the examiners will engage a discussion with the candidates to assess, despite this difficulty, their knowledge level on the topic.

S7 final baccalaureate mark in mathematics (criteria for evaluating the advanced mathematics oral exam)

The oral examination gives the pupils an opportunity to express themselves on a mathematical topic. Besides the validity of the answer, one will attach paramount importance to the basis of the argument and the relevance of justification without neglecting the quality of oral expression. Therefore, the matrix used to assess the pupil will take in consideration:

- The plan of the presentation: the pupils must show that the topic to which the question refers is familiar and justify the approach they will implement. Specifically, they must:
    - identify the topic;
    - clarify the concepts and methods being implemented;
    - show the ability to set the given problem in a mathematical context.
- The development of the solution: during the solution of the question the pupils must:
    - recall necessary definitions;
    - use appropriate vocabulary;
    - show a consistent, logical approach;
    - show mastery of any computational techniques that are used (with or without a technological tool).

  Additional questions are not predetermined and depend on the quality of the presentation of the pupils. They are designed to:
    - assess the knowledge level of pupils on the topic of the question chosen (mainly if the pupils' development can be improved);
    - broaden the question (extrapolation).
- Practical requirements: during the solution of the question they will also be evaluated on the following:
    - clear communication skills and use of an appropriate vocabulary;
    - good use and management of the black/white board;
    - ability to adapt to an oral examination.

5.1. Attainment Descriptors

	A	B	C	D	E	F	FX
	(9,0 - 10 Excellent)	(8,0 - 8,9 Very good)	(7,0 - 7,9 Good)	(6,0 - 6,9 Satisfactory)	(5,0 - 5,9 Sufficient)	(3,0 - 4,9 Weak/Failed)	(0 - 2,9 Very weak/Failed)
Knowledge and comprehension	Demonstrates comprehensive knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows broad knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in most areas of the programme	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Shows partial knowledge and limited understanding of mathematical terms, symbols, and principles	Shows very little knowledge and understanding of mathematical terms, symbols and principles
Methods	Successfully carries out mathematical processes in all areas of the syllabus	Successfully carries out mathematical processes in most areas of the syllabus	Successfully carries out mathematical processes in a variety of contexts	Successfully carries out mathematical processes in straightforward contexts	Carries out mathematical processes in straightforward contexts, but with some errors	Carries out mathematical processes in straightforward contexts, but makes frequent errors	Does not carry out appropriate processes
Problem solving	Translates complex non-routine problems into mathematical symbols and reasons to a correct result; makes and uses connections between different parts of the programme	Translates non-routine problems into mathematical symbols and reasons to a correct result; makes some connections between different parts of the programme	Translates routine problems into mathematical symbols and reasons to a correct result	Translates routine problems into mathematical symbols and reasons to a result	Translates routine problems into mathematical symbols and attempts to reason to a result	N/A	N/A

Interpretation	Draws full and relevant conclusions from information; evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information, evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information and attempts to evaluate reasonableness of results	Attempts to draw conclusions from information given, shows some understanding of the reasonableness of results	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Makes little attempt to interpret information	N/A
Communication	Consistently presents reasoning and results in a clear, effective and concise manner, using mathematical terminology and notation correctly	Consistently presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results adequately using mathematical terminology and notation	Generally presents reasoning and results adequately using some mathematical terminology and notation	Attempts to present reasoning and results using mathematical terms	Displays insufficient reasoning and use of mathematical terms
Digital competence	Uses technology appropriately and creatively in a wide range of situations	Uses technology appropriately in a wide range of situations	Uses technology appropriately most of the time	Uses technology satisfactorily most of the time	Uses technology satisfactorily in straightforward situations	Uses technology to a limited extent	Does not use technology satisfactorily

5.2. Oral examination Advanced Mathematics - Assessment record sheet

European school …………………… Pupil: ……………………………………… Class: ……….. Examiner: ………………………………………

	(9,0 - 10 Excellent)	(8,0 - 8,9 Very good)	(7,0 - 7,9 Good)	(6,0 - 6,9 Satisfactory)	(5,0 - 5,9 Sufficient)	(3,0 - 4,9 Failed/Weak)	(0 - 2,9 Failed/Very weak)	**Mark**
Knowledge and comprehension	Demonstrates comprehensive knowledge and understanding of mathematical terms, symbols and procedures relevant to the question	Shows broad knowledge and understanding of mathematical terms, symbols and procedures relevant to the question	Shows satisfactory knowledge and understanding of mathematical terms, symbols and procedures relevant to the question	Shows satisfactory knowledge and understanding of mathematical terms, symbols and procedures relevant to the question, possibly with some prompting	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and procedures relevant to the question, but with some errors and prompting	Shows partial knowledge and limited understanding of mathematical terms, symbols, and procedures relevant to the question, but makes frequent errors and requires frequent prompting	Shows very little knowledge and understanding of mathematical terms, symbols and procedures relevant to the question	
Methods	Successfully carries out mathematical processes in all parts of the question	Successfully carries out mathematical processes in a variety of contexts	Successfully carries out mathematical processes in a variety of contexts, possibly with some prompting	Successfully carries out mathematical processes in straightforward contexts, possibly with some prompting	Carries out mathematical processes in straightforward contexts, but with some errors and prompting	Carries out mathematical processes in straightforward contexts but makes frequent errors and requires frequent prompting	Does not carry out appropriate processes	
Problem solving	Translates complex non-routine problems into mathematical symbols and reasons to a correct result; potentially makes and uses connections between different parts of the programme if	Translates non-routine problems into mathematical symbols and reasons to a correct result; makes some connections between different parts of the programme if relevant to the question	Translates routine problems into mathematical symbols and reasons to a correct result	Translates routine problems into mathematical symbols and reasons to a result, possibly with some prompting	Translates routine problems into mathematical symbols and attempts to reason to a result, possibly with more extensive prompting	N/A	N/A	

	relevant to the question							
Interpretation	Draws full and relevant conclusions from information; evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information, evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information and attempts to evaluate reasonableness of results	Attempts to draw conclusions from information given, shows some understanding of the reasonableness of results	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Makes little attempt to interpret information	N/A	
Communication and planning	Consistently presents reasoning and results in a clear, effective and concise manner, using mathematical terminology and notation correctly	Consistently presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results adequately using mathematical terminology and notation	Generally presents reasoning and results adequately using some mathematical terminology and notation	Attempts to present reasoning and results using mathematical terms	Displays insufficient reasoning and use of mathematical terms	
Technology and tools (if applicable)	Uses technology and/or other presentation tools appropriately and creatively in a wide range of situations	Uses technology and/or other presentation tools appropriately in a wide range of situations	Uses technology and/or other presentation tools appropriately most of the time	Uses technology and/or other presentation tools satisfactorily most of the time	Uses technology and/or other presentation tools satisfactorily in straightforward situations	Uses technology and/or other presentation tools to a limited extent	Does not use technology and/or other presentation tools satisfactorily	
							Global Mark	

Date: ……………………… Signature: ………………………………………………

Annex 1: Suggested time frame

For this cycle, the following topics are described with only an estimated number of weeks to be reviewed by the teacher depending on the class.

With regard to the order of the topics, the teachers should bear in mind that Analysis must be dealt with at a later stage and only when differentiation has been covered in the S6MA5 course. Similarly, in S7 the Analysis content relies on the integration taught in the S7MA5 course. It would therefore be advisable to follow the order of the suggested time frame in S6. As for S7, an option would be to start with Linear algebra and continue with one or two optional topics in S7, before working on the Analysis content.

Note: The designated weeks include assessments, time needed for practice and rehearsal, mathematics projects, school projects, etcetera.

Course	**S6AM**	**S7AM**
Topic	**Weeks**	
Foundations of mathematics	9	/
Number theory	9	/
Linear algebra	9	5
Analysis	3	9
1st optional topic	/	7
2nd optional topic	/	7
Total	**30**	**28**