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Pedagogical Development Unit

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Mathematics Syllabus – S4-S5[1]
4 Periods (4P)

Approved by the Joint Teaching Committee at its meeting of 7 and 8 February 2019 in Brussels

Entry into force: on 1 September 2019 for S4
on 1 September 2020 for S5

[1] The document "Mathematics Syllabus – S5 - 4 Periods" (ref. 2019-01-D-48-en-3) was approved by the Joint Teaching Committee at its meeting of 13 and 14 February 2020 in Brussels

Europeans Schools - Mathematics Syllabus

Year S4-S5 – 4 P

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1. General Objectives

The European Schools have the two objectives of providing formal education and of encouraging pupils' personal development in a wider social and cultural context. Formal education involves the acquisition of competences (knowledge, skills and attitudes) across a range of domains. Personal development takes place in a variety of spiritual, moral, social and cultural contexts. It involves an awareness of appropriate behaviour, an understanding of the environment in which pupils live, and a development of their individual identity.

These two objectives are nurtured in the context of an enhanced awareness of the richness of European culture. Awareness and experience of a shared European life should lead pupils towards a greater respect for the traditions of each individual country and region in Europe, while developing and preserving their own national identities.

The pupils of the European Schools are future citizens of Europe and the world. As such, they need a range of competences if they are to meet the challenges of a rapidly-changing world. In 2006 the European Council and European Parliament adopted a European Framework for Key Competences for Lifelong Learning. It identifies eight key competences which all individuals need for personal fulfilment and development, for active citizenship, for social inclusion and for employment:

1. Literacy competence
2. Multilingual competence
3. Mathematical competence and competence in science, technology and engineering
4. Digital competence
5. Personal, social and learning to learn competence
6. Civic competence
7. Entrepreneurship competence
8. Cultural awareness and expression competence

The European Schools' syllabuses seek to develop all of these key competences in the pupils.

Key competences are that general, that we do not mention them all the time in the Science and Mathematics syllabuses.

2. Didactical Principles

General

In the description of the learning objectives, competences, connected to content, play an important role. This position in the learning objectives reflects the importance of competences acquisition in actual education. Exploratory activities by pupils support this acquisition of competences, such as in experimenting, designing, searching for explanations and discussing with peers and teachers. In science education, a teaching approach is recommended that helps pupils to get acquainted with concepts by having them observe, investigate and explain phenomena, followed by the step to have them make abstractions and models. In mathematics education, investigations, making abstractions and modelling are equally important. In these approaches, it is essential that a maximum of activity by pupils themselves is stimulated – not to be confused with an absent teacher: teacher guidance is an essential contribution to targeted stimulation of pupils' activities.

The concept of *inquiry-based learning* (IBL) refers to these approaches. An overview of useful literature on this can be found in the *PRIMAS guide for professional development providers*.

http://primas-project.eu/wp-content/uploads/sites/323/2017/10/PRIMAS_Guide-for-Professional-Development-Providers-IBL_110510.pdf

Mathematics

Careful thought has been given to the content and the structure to where topics are first met in a pupil’s time learning mathematics in secondary education. It is believed that this is a journey and if too much content is met at one point, there is a risk that it will not be adequately understood and thus a general mathematical concept will not be fully appreciated. By limiting the content of this syllabus (found in section 4.2.) each year more time can be used to develop core mathematical concepts that may have been met before or new mathematical concepts introduced are given ample time for extension. It must be noted that extension activities are conducted at the discretion of the teacher, however, it is suggested that rather than look at a vertical approach to extension a horizontal approach is used, thus giving the pupil a deeper understanding of the mathematical concept (in section 4 the word ‘limitation’ is used to ensure the extension does not go too far).

Furthermore, to this point it is believed that with a focus on competences this syllabus can encourage pupils to have a greater enjoyment of mathematics, as they not only understand the content better but understand the historical context (where it is expected a history of mathematics can be told over the cycles) and how the mathematics can be applied in other subjects, cross cutting (these can be seen in the fourth column in section 4.2.). As such the syllabuses have specifically been designed with reflection to the key competences (section 1) and the subject specific competences (section 3.1). In some cases, the key competences are clear for example the numerous history suggested activities (shown by the icon ) that maps to key competency 8 (Cultural awareness and expression). In other areas the link may not be so apparent.

One of the tasks in the pupil’s learning process is developing inference skills, analytical skills and strategic thinking, which are linked to both the key and subject specific competences. This is the ability to plan further steps in order to succeed solving a problem as well as dividing the process of solving more complex problems into smaller steps. A goal of teaching mathematics is to develop pupil’s intuitions in mathematics appropriate for their age. The ability to understand and use mathematical concepts (e.g. angle, length, area, formulae and equations) is much more important than memorising formal definitions.

This syllabus has also been written so that it can be accessible by teachers, parents and pupils. This is one reason why icons have been used (listed in section 4.2.). These icons represent different areas of mathematics and are not necessarily connected to just one competency but can cover a number of competences.

To ensure pupils have a good understanding of the mathematics the courses from S1 to S7 have been developed linearly with each year the work from the previous year is used as a foundation to build onto. Thus, it is essential before commencing a year the preceding course must have been covered or a course that is similar. The teacher is in the best position to understand the specific needs of the class and before beginning a particular topic it is expected that pupils have the pre-required knowledge. A refresh is always a good idea when meeting a concept for the first time in a while. It should be noted that revision is not included in the syllabus, however, as mentioned earlier about limiting new content, there is time to do this when needed.

The use of technology and digital tools plays an important role in both theoretical and applied mathematics, which is reflected in this syllabus. The pupils should get the opportunity to work and solve problems with different tools such as spreadsheets, computer algebra system (CAS) software, dynamic geometric software (DGS), programming software or other software that are available in the respective schools. Technology and digital tools should be used to support and promote pupils' understanding, for example by visualising difficult concepts and providing interactive and personalised learning opportunities, rather than as a substitute for understanding. Their use will also lead to improved digital competence.

Teachers have full discretion with how to teach this course, materials to use and even the sequence the content is taught in. The content and the competencies (indicated in the tables in section 4.2., columns 2 and 3) to be covered is, however, mandatory.

The S4 4 Period Course

The S4 4 Period course has been developed alongside the 6 Period course where the core work is done in the 4 Period course the 6 Period course will explore the content in more depth. With this approach changing between the courses is possible with the understanding that pupils having studied the 6 Period course will often have a greater depth of understanding.

The S4 4 Period course builds upon the ground work of the S1, S2 and S3 courses. However, there is a particular focus on Algebra, Geometry and Statistics and Probability. Indeed, this is the first year that the Probability topic is met and although there is no specific Number topic in this course plenty of the content encountered in the Number topic of the S1, S2 and S3 courses will be required for example the use of fractions for Probability.

The S5 4 Period Course

This course has been specifically written for those that are not choosing mathematics at a higher level. The aim of the course is less interested in why the mathematics works but finding models that can be applied to satisfy given criteria. It is hoped that by following this applied approach pupils that have historically had difficulties with theoretical mathematics should have a better understanding of the application of the topics met. Hence this course focuses on modelling and in section 5.2. of this document the rationale and an example of how to use modelling is explained. This has applications in science, economics, geography and beyond with an aim of promoting life-long learning. The use of mathematics in a cross curricula setting has been at the foremost point in developing this syllabus.

Pupils must note that the 4 Period and 6 Period courses in S5 are different. Thus, pupils wishing to study the 5 Period course in S6 will need to be aware of this before embarking on the 4 Period course in not just S5 but S4 too.

3. Learning Objectives

3.1. Competences

The following are the list of subject specific competences for mathematics. Here the key vocabulary is listed so that when it comes to reading the tables in section 4.2. the competency being assessed can be quickly seen. Please note that the list of key vocabulary is not exhaustive, and the same word can apply to more than one competency depending on the context.

Further information about assessing the level of competences can be found in section 5.1. Attainment Descriptors. The key concepts here are those needed to attain a sufficient mark.

	Competency	Key concepts (attain 5.0-5.9)	Key vocabulary
1.	**Knowledge and comprehension**	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Apply, classify, compare, convert, define, determine, distinguish, expand, express, factorise, identify, know, manipulate, name, order, prove, recall, recognise, round, simplify, understand, verify
2.	**Methods**	Carries out mathematical processes in straightforward contexts, but with some errors	Apply, calculate, construct, convert, draw, manipulate model, organise, plot, show, simplify sketch solve, use, verify
3.	**Problem solving**	Translates routine problems into mathematical symbols and attempts to reason to a result	Analyse, classify, compare, create, develop, display, estimate, generate, interpret, investigate, measure, model, represent, round, simplify, solve
4.	**Interpretation**	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Calculate, conduct, create, develop, discover, display, generate, interpret, investigate, model
5.	**Communication**	Generally presents reasoning and results adequately; using some mathematical terminology and notation	Calculate, conduct, create, discover, display, interpret, investigate, model, present
6.	**Digital competence**[2]	Uses technology satisfactorily in straightforward situations	Calculate, construct, create, display, draw, model, plot, present, solve

[2] This competence is part of the European Digital Competence Framework (https://ex.europa.eu/jrc/en/digcomp)

3.2. Cross-cutting concepts

Cross cutting concepts will be carried by the joint competences. The list of cross cutting concepts that will be composed will be shared by all science and mathematics syllabuses. The tentative list to be taught is based on the next generation science standards in the United states (National Research Council, 2013):

	Concept	Description
1.	**Patterns**	Observed patterns of forms and events guide organisation and classification, and they prompt questions about relationships and the factors that influence them.
2.	**Cause and effect**	Mechanism and explanation. Events have causes, sometimes simple, sometimes multifaceted. A major activity of science is investigating and explaining causal relationships and the mechanisms by which they are mediated. Such mechanisms can then be tested across given contexts and used to predict and explain events in new contexts.
3.	**Scale, proportion and quantity**	In considering phenomena, it is critical to recognise what is relevant at different measures of size, time, and energy and to recognise how changes in scale, proportion, or quantity affect a system's structure or performance.
4.	**Systems and system models**	Defining the system under study—specifying its boundaries and making explicit a model of that system—provides tools for understanding the world. Often, systems can be divided into subsystems and systems can be combined into larger systems depending on the question of interest
5.	**Flows, cycles and conservation**	Tracking fluxes of energy and matter into, out of, and within systems helps one understand the systems' possibilities and limitations.
6.	**Structure and function**	The way in which an object or living thing is shaped and its substructure determine many of its properties and functions and vice versa.
7.	**Stability and change**	For natural and built systems alike, conditions of stability and determinants of rates of change or evolution of a system are critical for its behaviour and therefore worth studying.
8.	**Nature of Science**	All science relies on a number of basic concepts, like the necessity of empirical proof and the process of peer review.
9.	**Value thinking**	Values thinking involves concepts of justice, equity, social–ecological integrity and ethics within the application of scientific knowledge.

In the mathematics syllabuses, the concepts 5 and 8 will be addressed only to a limited extent. The lists of competences and cross cutting concepts will serve as a main cross-curricular binding mechanism. The subtopics within the individual syllabuses will refer to these two aspects by linking to them in the learning goals.

http://ngss.nsta.org/Professional-Learning.aspx

4. Content

4.1. Topics

This section contains the tables with the learning objectives and the mandatory content for the strand Mathematics in S4 (4 periods per week).

4.2. Tables

How to read the tables on the following pages

The learning objectives are the curriculum goals. They are described in the third column. These include the key vocabulary, highlighted in bold, that are linked to the specific mathematics competences found in section 3.1. of this document.

These goals are related to content and to competences. The mandatory content is described in the second column. The final column is used for suggested activities, key contexts and phenomena. The teacher is free to use these suggestions or use their own providing that the learning objective and competencies have been met.

Please note that the word ‘limitation’ is used to ensure that when extension is planned it is planned with the idea of horizontal extension rather than vertical extension as mentioned in section 2. of this document.

Use of icons

Furthermore, there are six different icons which indicate the areas met in the final column:

	Activity
	Cross-cutting concepts
	Digital competence
	Extension
	History
	Phenomenon

Each of these icons highlight a different area and are used to make the syllabus easier to read. These areas are based on the key competences mentioned in section 1 of this document.

S4 – 4 Period (4P)

YEAR 4 (4P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Basic calculations	Basic calculations (+, –, ×, /)	**Apply** basic calculations (+, –, ×, /) over the set of $\mathbb{Q}$	Calculations are used in all sciences A common approach on how to calculate and how to use calculators is highly recommended
	Calculation rules and properties	**Verify** calculation rules and properties and use them in simple algebraic expressions	Origins of numbers, e.g. Egyptian fractions
	Fractions Decimal numbers Approximate and exact mode LCM and HCF Simplification and factorisation	**Use** a technological tool or a software to: • **transform** a fraction into decimal numbers and vice versa • **manage** approximate and exact mode calculation • **calculate** Lowest Common Multiple (LCM) and the Highest Common Factor (HCF) • **simplify** and **factorise** numerical and algebraic expressions • **check** results	
Square numbers, square roots and powers	Powers	**Use** calculation rules and properties established in years 1 to 3	
	Square numbers	**Recall** the first 20 square numbers	Happy numbers

YEAR 4 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Squaring and square root as inverse operations	**Understand** that squaring and square rooting are inverse operations		
Radicals and surds	Surds	**Understand** that $\sqrt{2} \notin \mathbb{Q}$ and **recognise** other surds **Know** the distinction between exact and approximate calculations		In the ISO paper size system, the height-to-width ratio of all pages is equal to the square root of two Appreciate that in other scientific subjects, pupils are expected to manipulate with roots and squares
	Properties of radicals	**Apply** the following properties of radicals: • $\sqrt{a}\sqrt{b} = \sqrt{ab}$ for $a, b \in \mathbb{R}^+ \cup \{0\}$ • $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ for $a \in \mathbb{R}^+ \cup \{0\}, b \in \mathbb{R}^+$ • $\sqrt{a^2 b} = a\sqrt{b}$ for $a, b \in \mathbb{N}$ • $\sqrt{a^2} = \lvert a \rvert$ for $a \in \mathbb{R}$		
	Rationalise a denominator	**Simplify** surds by rationalising the denominator limited to division by $\sqrt{a}$ for $a \in \mathbb{N}$		Investigate why the denominator is rationalised linking in with adding and subtracting fractions
Proportionality	Direct proportion	**Investigate** phenomena which can be modelled with direct proportion: $y = k \cdot x$		Use phenomena from different subjects with a focus on the similar underlying structure, e.g. $s = v \cdot t$ Use with calculating exchange rates, e.g. what is 10€ in US$?

YEAR 4 (4P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
	Inverse proportion	**Investigate** phenomena which can be modelled with inverse proportion: $y = \frac{k}{x}$	Investigate what happens at $x = 0$ and as x becomes large
		Use table of values	
	Representations of direct and inverse proportions	**Represent** direct and inverse proportions with graphs	Use appropriate technological tools to represent these graphs
			Use these graphs to help make it much easier to convert between different units
Linear models	Relations and functions	**Define** a relation and a function in establishing that one variable (quantity) is dependent on another variable (quantity)	Use examples of linear models understanding the difference between dependent and independent variables
		Understand the differences and similarities between relations and functions, e.g. use a vertical line test	Investigate real problems, e.g. from natural and social sciences, which can be modelled with linear models
		Use function notation ($y = f(x)$) and vocabulary with and without a technological tool	
		Understand and **apply** the following equations $ax + by + c = 0$ and $y = mx + p$ and **convert** from the first form to the second	Convert from the second form back to the first
	Variables and parameters	**Understand** the difference between variables and parameters	

YEAR 4 (4P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
	Linear graphs	**Represent** linear relations with graphs	
		Use technology to manipulate parameters and see the effect on linear graphs	Use appropriate CAS technology to investigate what changing the variables and parameters have in a function/model; it is particularly important that pupils understand the effect of changing the gradient on the function
		Calculate the axes intercepts for linear graphs and **solve** other equations that are related to linear formulae	Investigate how the area can be modelled by just varying one length
Simultaneous equations of the type: $\begin{cases} ax + by = c \\ dx + ey = f \end{cases}$	Linear equations	**Investigate** real problems which can be **modelled** with simultaneous linear equations	Use examples of simultaneous equations from natural and social sciences Possible chance to look into how algebra was introduced into mathematics and which part of the world it came from
	Simultaneous linear equations	**Solve** simultaneous linear equations by plotting the graphs and algebraically (by substitution and/or elimination)	Discuss limitations of each method
Polynomials	Polynomial expressions	**Simplify** algebraic expressions with powers and recognise equivalent expressions, e.g. $ax^n + bx^n = (a+b)x^n$, where $n \in \mathbb{N}$	

YEAR 4 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Understand that a polynomial is an expression consisting of variables and coefficients of the following forms: $ax^2 + bx + c$		
		Know how to add polynomial expressions in one variable, e.g. $(ax^2 + bx) + (cx^2 + d)$, where a, b, c and $d \in \mathbb{Z}$		Look at how to subtract and/or multiply polynomial expressions
				Use appropriate CAS technology to check results
		Know that the order of a polynomial is determined by the highest power of the variable		
		Simplify and **order** polynomial expressions with one variable		Polynomials with more than one variable
	Quadratic expressions	**Know** how to factorise the quadratic expressions: • $ax^2 + bx = x(ax + b)$ • $ax^2 + abx = ax(x + b)$ • $abx^2 + ax = ax(bx + 1)$		Factorise $x^2 + ax + bx + ab$ to the form $(x + a)(x + b)$

<table>
<tr><th>YEAR 4 (4P)</th><th colspan="3">TOPIC: ALGEBRA</th></tr>
<tr><th>Subtopic</th><th>Content</th><th>Learning objectives</th><th>Key contexts, phenomena and activities</th></tr>
<tr><td></td><td></td><td>Apply the special identities:
• $(a \pm b)^2 = a^2 \pm 2ab + b^2$
• $(a + b)(a - b) = a^2 - b^2$</td><td>Use of the area model to develop this topic first met in S3:
<table><tr><td>a^2</td><td>$a \cdot b$</td></tr><tr><td>$a \cdot b$</td><td>b^2</td></tr></table>
Use the special identities to help with calculations such as: $20 \cdot 20 = 400$, $19 \cdot 21 = 199$ or $18 \cdot 22 = 196$</td></tr>
</table>

YEAR 4 (4P)	TOPIC: GEOMETRY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Right-angled triangles	Pythagoras’ theorem	**Know** Pythagoras' theorem and its converse	Find and investigate different proofs of Pythagoras theorem
		Understand one proof of Pythagoras' theorem	The history of Pythagoras’ theorem is certainly worth exploring from looking at: • Babylonians first using a rule • Idea of irrational numbers and $\sqrt{2}$ driving Pythagoras to mad • The idea of a proof that once something is proved it is true forever
		Solve real life problems by applying Pythagoras' theorem (calculate lengths and angles in right angled triangles**)** *Limitation: limit here to 2D*	• Calculate the length of a zipline • Calculate the distance between two points Investigate the comment the shortest distance between two points is a straight line
	Trigonometric ratios	**Define**, **recognise** and **apply** the trigonometric ratios sin, cos and tan to calculate lengths and angles in right-angled triangles	Angle of elevation and depression, $\tan x = \frac{\sin x}{\cos x}$
Enlargement	Geometrical enlargements and reductions	**Apply** geometrical enlargement and reduction (scale factor rational number) using a centre of enlargement	Photography, Matryoshka doll, …
	Scale factor of an enlargement	**Determine** the scale factor of an enlargement	Compare a model (car) with a real one

YEAR 4 (4P)	**TOPIC: GEOMETRY**			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Use software to find out the scale factor of an enlargement using variables and sliders		There are several ways of using technology; one could be to use the zoom function on a mobile phone and secondly use appropriate CAD/CAS tools with centres of enlargements on a coordinate plane
	Invariants of enlargements	**Understand** and **recognise** the invariants (angles, parallelism, ratios…) of enlargements		
Similar triangles	Congruent and similar triangles	**Recognise** congruent and similar triangles		Fractals, e.g. constructing Sierpinski triangle
	Intercept theorem	**Know, recognise** and **apply** the intercept theorem, e.g. enlargements		Apply the intercept theorem to prove lines are parallel
				Use software to demonstrate the intercept theorem using variables and sliders

YEAR 4 (4P)	TOPIC: STATISTICS AND PROBABILITY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
Data collection	Level of measurement Data interpretation	**Analyse** the level of measurement of a variable in a concrete situation, being either nominal, ordinal (both categorical), interval or ratio (both numerical)	Classify statistical data as they appear in news media according to their measurement levels
		Understand the value of data interpretation and the (mis)use of statistics (the goal is to develop a critical attitude)	Make pupils find misleading statistical representations Use data from scientific sources
Organise data	Absolute, relative and cumulative frequencies	**Understand** the meaning of absolute, relative and cumulative frequencies	Use data from scientific sources
		Calculate different types of frequencies (write relative frequencies as fractions and percentages) by hand and a spreadsheet software or other appropriate digital technology	Make pupils change frequency tables according to specific purposes
		Use relative frequencies to **compare** different data sets *Limitation: do not use too many data points and make pupils aware of the limitations of this representation*	Make the distinction between percentages and points of percentages
	Frequency table	**Organise data** in a frequency table, by hand and using spreadsheet software or other appropriate digital technology, including all of the above frequency types	

YEAR 4 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Stem-and-leaf diagram	**Organise** a small set ($N < 25$) of numerical data **i**n a stem-and-leaf plot		
Data set characteristics	Measures of centre	**Understand** the meaning of different measures of central tendency and know when to apply which measure appropriately and how to interpret its value		Different data sets should be used to highlight the strengths and weaknesses of each type of average • Look into how statistics can be manipulated and that how it can be used can be misleading • Investigate how averages are used in different sport and why they are important
	Mode	**Identify** and **interpret** the mode in an appropriate data set, also in case the data are presented in a frequency table		Research data sets and diagrams from Eurostat
	Mean	**Calculate** and **interpret** the (arithmetic) mean of a set of data at interval or ratio measurement level, also in case the data are presented in a frequency table		
	Median	**Determine** and **interpret** the median of a set of data also in case the data is presented in a frequency table		
	Quartiles	**Understand** the meaning and **determine** the quartiles of a set of data		
	Measures of spread	**Understand** the meaning and how to calculate the range		Make pupils compare the impact of outliers on the data set

YEAR 4 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Understand the meaning and how to calculate the interquartile range		
Graphical representations of data	Graphical representations	**Investigate** and **interpret** graphical representations **Use** graphical representations to find estimations of central tendency and dispersion by hand and using spreadsheet software or other appropriate digital technology *Note: make pupils aware of the difference between a histogram and a bar chart*		Make pupils chose between different possible representations of the same data set.
	Pie chart	**Interpret** a given pie chart **Present** a data set using a pie chart *Limitation: should be limited to no more than 6 categories (pupils should understand why having too many categories can make a pie chart difficult to interpret)*		When creating a pie chart that has several categories introduce the idea of an 'other' category thus now the concept of listing everything or 'exhaustive' lists so that every choice is included
	Bar chart	**Interpret** a given bar chart **Present** a data set using a bar chart		
	Histogram (frequency diagram)	**Interpret** a given histogram **Present** a set of grouped data at interval or ratio measurement level provided in a table with equal class width using a histogram		

<table>
<tr><th>YEAR 4 (4P)</th><th colspan="4">TOPIC: STATISTICS AND PROBABILITY</th></tr>
<tr><th>Subtopic</th><th>Content</th><th>Learning objectives</th><th colspan="2">Key contexts, phenomena and activities</th></tr>
<tr><td></td><td>Cumulative frequency polygon</td><td>Interpret a given cumulative frequency polygon

Present a set of data using a cumulative frequency polygon</td><td></td><td></td></tr>
<tr><td></td><td>Box plot</td><td>Interpret a given boxplot

Present a set of data using a box plot</td><td></td><td>It is important to note that some technological tools may highlight outliers automatically; a discussion of outliers should be made at this point</td></tr>
<tr><td></td><td>Comparing two sets of data</td><td>Use the above graphical representations to compare two sets of data with respect to key data points and the spread of the data</td><td></td><td></td></tr>
<tr><td>Probability</td><td>Sample space</td><td>Define the sample space in a random experiment
Note: make pupils aware that outcome sets may be infinite sets</td><td></td><td>Use a systematic listing technique to ensure that all outcomes have been included, e.g. two fair six-sided dice are rolled and their scores added; list all outcomes that there can be leads to:
<table>
<tr><th></th><th colspan="5">Dice 1</th></tr>
<tr><th></th><th></th><th>1</th><th>2</th><th>3</th><th>…</th></tr>
<tr><th rowspan="4">Dice 2</th><th>1</th><td>2</td><td>3</td><td>4</td><td>…</td></tr>
<tr><th>2</th><td>3</td><td>4</td><td>5</td><td>…</td></tr>
<tr><th>3</th><td>4</td><td>5</td><td>6</td><td>…</td></tr>
<tr><th>…</th><td>…</td><td>…</td><td>…</td><td>…</td></tr>
</table></td></tr>
<tr><td></td><td>Event</td><td>Understand that an event is a subset of the set of all possible outcomes</td><td></td><td></td></tr>
</table>

YEAR 4 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Venn diagram	**Use** a Venn diagram to **represent** the set of possible outcomes and events		Venn diagrams may have already be used with sets as they are an excellent visual way of classifying items
	Tree diagram	**Use** a tree diagram to **represent** the set of possible outcomes and events *Limitation: no more than three sets of branches in the tree diagram*		Tree diagrams are represented in many different ways across the different sections looking into how different language sections represent these diagrams may help pupils understand them better
	Concept of complementary, independent, mutually exclusive, and exhaustive events	**Understand** the idea of probability leading on from relative frequency **Calculate** probabilities using Venn diagrams and tree diagrams		Make pupils distinguish between experimental and theoretical probability, e.g. tossing a coin, rolling a die, …

S5 – 4 Period (4P)(

YEAR 5 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Powers	Negative and rational powers	**Understand** the meaning of negative and rational powers		Investigate relations with all kind of formulae from physics, chemistry and biology
		Understand the relation of rational powers and roots, e.g. $\sqrt{x} = x^{\frac{1}{2}}$		
		Use negative and rational powers to rewrite scientific formulae, e.g. $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\left(\frac{l}{g}\right)^{\frac{1}{2}}$ or $l = g\left(\frac{T}{2\pi}\right)^{2}$ *Note: formulae can be expressed in many ways using negative and rational powers instead of divisions and roots* *Limitation: powers in formulae:* $-1, -2, -3$, $\frac{1}{2}$, $\frac{1}{3}$ *and* $\frac{2}{3}$.		Calculate with units using negative exponents, e.g. $\frac{m}{s} = m.s^{-1}$
	Scientific notation	**Understand** how to write a number in scientific notation with positive and negative powers and how to convert a number from scientific notation to a number with and without the use of a technological tool		Translate between calculator notation and mathematic notation
		Apply SI-prefixes: • micro, nano, pico, femto • Giga, Tera, Peta, Exa		Relation with physics, chemistry and biology, e.g. Avogadro
		Calculate (add, subtract, multiply and divide) with scientific notation		Round the answer to a certain number of significant figures

YEAR 5 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
				Understand why we round with significant figures and not decimal places
				Make the difference between accuracy of measurement and precision
Polynomials	Quadratic models	**Investigate** the class of models $y = a \cdot x^2$ which describes quadratic processes		Investigate quadratic models from economics, physics, chemistry and biology
		Use an appropriate technological tool to **investigate** the graphs of $y = a(x-p)^2 + q$ by manipulating the parameters a, p and q.		Connect with function notation: $y = a \cdot f(x)$, $y = f(x-p)$ and $y = f(x) + q$
		Recognise that the following equations $y = a(x-p)^2 + q$ and $y = ax^2 + bx + c$ are two descriptions of the same function		
	Quadratic functions	**Use** an appropriate technological tool to solve quadratic equations		
		Understand the required approach to **determine** the key points including axes-intercepts, the vertex and the axis of symmetry of a quadratic function		
		Apply these concepts to solve real life problems, e.g. trajectories (fountain, waterjets, projectiles, …)		See section 5.2 of this document for one example

YEAR 5 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Polynomials order greater than 2	**Recognise** polynomial expressions and calculate their value **Use** algebraic expressions with powers and **recognise** equivalent expressions **Know** how to add and multiply algebraic expressions with powers **Know** how to simplify and order polynomial expressions **Know** how to add and multiply polynomials in one variable **Apply** the special identities $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$		Use examples of formulae with powers from natural and social sciences, e.g. accelerated motion
		Investigate the class of **models** which describes a power relation, i.e. real problems which can be modelled with formulae with powers of the variable (or polynomials) **Investigate** graphs and tables using an appropriate technological tool **Solve** simple **equations** algebraically (single power, quadratic equations) **Solve equations** graphically (with graphing software) or with CAS **Know** how to expand formulae with products, simple ones algebraically		Make pupils familiar with as many different relevant formulae from other fields as possible

YEAR 5 (4P)	TOPIC: ALGEBRA		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
		and complex ones by CAS	
	Pascal's triangle	**Investigate** the expansion of $y = (x+1)^n$ and relate to Pascal's triangle	History of Pascal's triangle with examples from India, Persia/Iran, China, Germany, and/or Italy
Exponential growth and decay: models and formulae	Exponential growth and decay $y = C \cdot a^x$	**Investigate** the class of models which describes exponential growth and decay	Investigate exponential models from economics, physics, chemistry and biology (compound interest, cell division, radioactive decay, decay-equations, …)
		Use the standard model $y = C \cdot a^x$ ($C > 0$, $a > 0$ and $a \neq 1$), **explain** how a affects growth or decay and **link** with percentages	Rice on chessboard (The Emperor of China)
		Compare linear and exponential models	
	Exponential equations	**Solve** exponential equations numerically or graphically by using a technological tool	Methods to solve include trial and improvement, using a spreadsheet, graph or using CAS
Periodic Models	Unit circle - Trigonometric functions	**Investigate** the unit circle and trigonometric functions (sin and cos) on the unit circle *Note: only after the introduction of units of angles in geometry; the trigonometric ratios were first used in S4; a review would be very useful*	Use and show periodic models from physics (waves) and biology (prey/predator), day length, sound, …
		Sketch the trigonometric functions $y = \sin x$, $y = \cos x$ and $y = \tan x$ over the domain of definition for one period and show that they are periodic	• **Use** an appropriate technological tool to **investigate** the graphs of the trigonometric functions, e.g. $y = a \cdot \sin(b(x+c)) + d$ by manipulating the parameters

YEAR 5 (4P)	TOPIC: ALGEBRA			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
				• Trigonometric models for the day length or the tides
Algorithm and program notions	Simple programming	**Use** a software to break down a problem into sub-problems, and write, test and execute a simple program **Create** a flow chart for basic algorithm **Know** how to assign labels to variables in a program **Understand** and **apply** different types of conditional instructions **Understand** and **apply** different types of computer loops		Approximate a square root: Heron's algorithm

YEAR 5 (4P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Units of angles	Degrees Radians	**Apply** the unit circle to **explain** how radians are defined **Use** the two different units (degrees and radians) to describe the magnitude of an angle **Convert** degrees to radians and vice-versa **Understand** how to derive the trigonometric ratios for a set of standard angles in degrees and radians: • $0°$, $30°$, $45°$, $60°$ and $90°$ • 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ radians		Historical concepts about radians
Solid Geometry	Length and formulae	**Apply** • Pythagoras' theorem $a^2 + b^2 = c^2$ (in two perpendicular planes) • Trigonometry • Intercept theorem to plane sections of solids to solve problems regarding the length, e.g. internal diagonal of a cube or cuboid, the edges of a pyramid or the height of a cone with particular angles		Pythagoras' Theorem in 3D: $a^2 + b^2 + c^2 = d^2$ Investigate what is the longest item that can fit into a pencil case/why a table when going through a door way needs to be tilted to fit through

YEAR 5 (4P)	TOPIC: GEOMETRY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Surface areas and formulae	**Recall** and **apply** appropriate formula to calculate surface areas of solids restricted to prisms, cylinders, pyramids, cones and spheres		Repeating nets from years 1 to 3 and calculating surface areas of examples from real life: church towers, houses, containers, …
	Volumes and formulae	**Understand** the effect on volume of enlargement on changing the scale		Change the volume, how does it change the radius or change parameters keeping the volume fixed
		Recognise and **solve** real problems which can be modelled with regular solids		Find as many examples from real life which can be modelled with regular solids (church towers, houses, containers, heating or air conditioning, (truncated) ice cream cone, …) and make calculations with them

YEAR 5 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
Probability	Classical probability	**Calculate** probability using the classical definition $\text{P}(A) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$		
	Complementary, mutually exclusive and exhaustive events	**Understand** how the probability formulae relate to Venn diagrams, tree diagrams and contingency tables		
		Calculate elementary probabilities using the complement of an event, mutually exclusive events (or non) and exhaustive events as well as using the following formulae: • $\text{P}(\bar{A}) = 1 - \text{P}(A)$ • $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B)$ for $A \cap B = \emptyset$ • $\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$ for $A \cap B \neq \emptyset$		Visualise formulae with a Venn diagrams
	Conditional probability	**Understand** the concept of conditional probability and the notations $\text{P}(A)$ and $\text{P}(A\|B)$		
		Use information from Venn diagrams, tree diagrams, contingency tables and the formula $\text{P}_B(A) = \text{P}(A\|B) = \frac{\text{P}(A \cap B)}{\text{P}(B)}$ to calculate conditional probability		
	Independent probability	**Understand** the concept of independent probability		

YEAR 5 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
		Use the formula $\mathrm{P}(A \cap B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$ or $\mathrm{P}_B(A) = \mathrm{P}(A\|B) = \mathrm{P}(A)$ to check if two events are independent		
Data collection	Surveys	**Construct** a survey to collect information		
		Understand the effect of bias on surveys, leading questions and the importance of exhaustive answers		Compare good and bad surveys
				Discuss how online media can manipulate opinion and be misused e.g. targeted advertising
	Representing surveys results	**Create** appropriate diagrams to represent the results of surveys		Calculate the mean, mode and median
				Effect of choice of scale to represent data
Sampling	Simulate sampling	**Understand** that different samples will show variation		
		Use a digital tool to simulate statistical sampling and interpret the results		Make pupils really experience sample variation
	Population Random sample	**Recognise** populations and random samples in everyday life situations and explain the difference between the two		How to make a representative committee
Inference	Statistical inference	**Recognise** that statistical inference concerns making claims about a population based on a sample		Investigate the statement “we can never be sure about these types of inferences”, taking into account the uncertainty of these generalisations

YEAR 5 (4P)	TOPIC: STATISTICS AND PROBABILITY		
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**
		Use stratified sampling techniques so that a sample better reflects the population compared to random sample **Make conclusions** by generalising findings from a sample to an underlying population	Discuss the merits of various sampling techniques • How is the European Parliament made up? Is it representative? • Is each member state proportionally represented at the Parliament? In the Commission? Compare to other system (electoral college in US)
Data Set Characteristics	Measure of variation Standard deviation	**Understand** that the standard deviation is a measure of variation and how it relates to the mean **Understand** the difference between the standard deviation for a population and a sample **Calculate** a standard deviation using a data table that leads to formula for a population standard deviation *Limitation: a data set of up to six values* **Using** spreadsheet software or other appropriate digital technology to determine the standard deviation for a population or a sample	Discuss how extreme values affect different measures of variation e.g. standard deviation and range Pupils collect data like body height, number of pens in their case, … Investigate the distribution of the population in the intervals $[\bar{x}-\sigma;\ \bar{x}+\sigma]$, $[\bar{x}-2\sigma;\ \bar{x}+2\sigma]$ and $[\bar{x}-3\sigma;\ \bar{x}+3\sigma]$

YEAR 5 (4P)	TOPIC: STATISTICS AND PROBABILITY			
Subtopic	**Content**	**Learning objectives**	**Key contexts, phenomena and activities**	
	Comparing data sets	**Use** the mean, mode, median, range, interquartile range and standard deviation to compare data sets *Limitation: only use these measures for non-categorical data*		

5. Assessment

For each level there are attainment descriptors written by the competences, which give an idea of the level that pupils have to reach. They also give an idea of the kind of assessments that can be done.

With the competences are verbs that give an idea of what kind of assessment can be used to assess that goal. In the table with learning objectives these verbs are used and put bold, so there is a direct link between the competences and the learning objectives.

Assessing content knowledge can be done by written questions where the pupil has to respond on. Partly that can be done by multiple choice but competences as constructing explanations and engaging in argument as well as the key competences as communication and mathematical competence need open questions or other ways of assessing.

An assignment where pupils have to use their factual knowledge to make an article or poster about a (broader) subject can be used to also judge the ability to critically analyse data and use concepts in unfamiliar situations and communicate logically and concisely about the subject.

In Europe (and America) pupils must have some competence in designing and/or engineering (STEM education). So there has to be an assessment that shows the ability in designing and communicating. A design assessment can also show the ability in teamwork.

Pupils have to be able to do an (experimental) inquiry. An (open) inquiry should be part of the assessments. Assessing designing and inquiry can be combined with other subjects or done by one subject, so pupils are not obliged to do too many designing or open inquiry just for assessment at the end of a year.

Digital competence can be assessed by working with spreadsheets, gathering information from internet, measuring data with measuring programs and hardware, modelling theory on the computer and comparing the outcomes of a model with measured data. Do combine this with other assessments where this competence is needed.

Assessment is formative when either formal or informal procedures are used to gather evidence of learning during the learning process and are used to adapt teaching to meet student needs. The process permits teachers and students to collect information about student progress and to suggest adjustments to the teacher's approach to instruction and the student's approach to learning.

Assessment is summative when it is used to evaluate student learning at the end of the instructional process or of a period of learning. The purpose is to summarise the students' achievements and to determine whether, and to what degree, the students have demonstrated understanding of that learning.

For all assessment, the marking scale of the European schools shall be used, as described in "*Marking system of the European schools: Guidelines for use*" (Ref.: 2017-05-D-29-en-7).

5.1. Attainment Descriptors

	A	B	C	D	E	F	FX
	(9,0 - 10 Excellent)	(8,0 - 8,9 Very good)	(7,0 - 7,9 Good)	(6,0 - 6,9 Satisfactory)	(5,0 - 5,9 Sufficient)	(3,0 - 4,9 Failed / Weak)	(0 - 2,9 Failed / Very weak)
Knowledge and comprehension	Demonstrates comprehensive knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows broad knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in all areas of the programme	Shows satisfactory knowledge and understanding of mathematical terms, symbols and principles in most areas of the programme	Demonstrates satisfactory knowledge and understanding of straightforward mathematical terms, symbols and principles	Shows partial knowledge and limited understanding of mathematical terms, symbols, and principles	Shows very little knowledge and understanding of mathematical terms, symbols and principles
Methods	Successfully carries out mathematical processes in all areas of the syllabus	Successfully carries out mathematical processes in most areas of the syllabus	Successfully carries out mathematical processes in a variety of contexts	Successfully carries out mathematical processes in straightforward contexts	Carries out mathematical processes in straightforward contexts, but with some errors	Carries out mathematical processes in straightforward contexts, but makes frequent errors	Does not carry out appropriate processes
Problem solving	Translates complex non-routine problems into mathematical symbols and reasons to a correct result; makes and uses connections between different parts of the programme	Translates non-routine problems into mathematical symbols and reasons to a correct result; makes some connections between different parts of the programme	Translates routine problems into mathematical symbols and reasons to a correct result	Translates routine problems into mathematical symbols and reasons to a result	Translates routine problems into mathematical symbols and attempts to reason to a result	Translates routine problems into mathematical symbols and attempts to reason to a result only with help	Does not translate routine problems into mathematical symbols nor attempts to reason to a result

Interpretation	Draws full and relevant conclusions from information; evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information, evaluates reasonableness of results and recognises own errors	Draws relevant conclusions from information and attempts to evaluate reasonableness of results	Attempts to draw conclusions from information given, shows some understanding of the reasonableness of results	Attempts to draw conclusions from information and shows limited understanding of the reasonableness of results	Makes little attempt to interpret information	Does not interpret information
Communication	Consistently presents reasoning and results in a clear, effective and concise manner, using mathematical terminology and notation correctly	Consistently presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results clearly using mathematical terminology and notation correctly	Generally presents reasoning and results adequately using mathematical terminology and notation	Generally presents reasoning and results adequately; using some mathematical terminology and notation	Attempts to present reasoning and results using mathematical terms	Displays insufficient reasoning and use of mathematical terms
Digital competence	Uses technology appropriately and creatively in a wide range of situations	Uses technology appropriately in a wide range of situations	Uses technology appropriately most of the time	Uses technology satisfactorily most of the time	Uses technology satisfactorily in straightforward situations	Uses technology to a limited extent	Does not use technology satisfactorily

Annex 1: Suggested time frame

For this cycle, the following topics are described with only an estimated number of weeks to be reviewed by the teacher depending on the class.

Note: The designated weeks include assessments, time needed for practice and rehearsal, mathematics projects, school projects, etcetera.

Course	**S4P4**	**S5P4**
Topic	**Weeks**	
Algebra	14	11
Geometry	8	8
Statistics and Probability	10	11
Total	**32**	**30**

Annex 2: Modelling

Physicists know very well, that it requires mathematics to describe the world. But also, many other scientific subjects like chemistry, economics or physical education use mathematical knowledge to explain phenomena or predict results. Above this, we live in the digital age and artificial intelligence will become a very important part of our lives.

When we talk about applied mathematics, we will find many examples, where problems are solved by using a modelling circle (see Figure 1). The weather forecast is such an example that comes up in our daily lives.

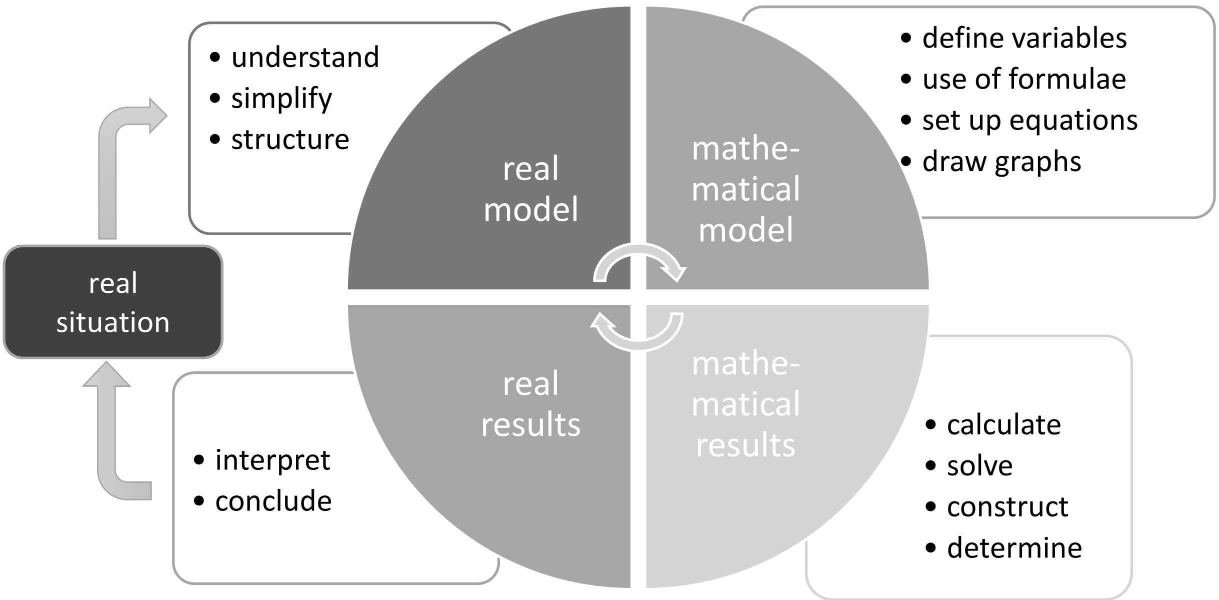

Figure 1: Modelling Circle (Borromeo-Ferri, 2006)

In this case, the *real situation* consists of data about the present weather and its development within the days before.

To create a *real model* for the weather, we need to understand for example, how humidity is related to the temperature. The weather is a very complex phenomenon, so we need to simplify and structure it before we can move on to the next step.

The *mathematical model* is based on many different equations that contain for example derivatives, Integrals and random variables.

To find *mathematical results* we need very fast computers that provide lots of data.

The *real results* can be found by interpreting the data and draw conclusions.

After going through the first circle the same procedure will be done again with slightly different results. So, in the end we can say for example, that the probability of rain for the next day is 20% because in 2 out of 10 trials this was our result.

One day later the results can be compared with the real situation to improve our model for the future. That the prediction is not always right shows, that we are dealing with a model. But when we look back for 10 years it is obvious, that the model has been optimised constantly.

When artificial intelligence is learning, the modelling circle plays an important role. In the 5 Period and the 6 Period courses we have pupils who might study informatics and develop new devices in the future. So, it is necessary that they are trained well in the complete circle. In the 3 Period and 4 Period courses we have pupils who use these devices. Therefore, it is important for them to understand, how these devices work in general. They should be able to question these devices and be critical towards the results given to them.

The modelling circle is quite complex itself, so it is recommended, that pupils are trained on single steps before putting them together or even undertake more than one stage. For the teacher it is possible to create tasks and exercises. The following example is to demonstrate how an existing question can be extended so that it even allows differentiation within the course.

Example of real situation: a sprinkler is used to water the garden.

1. To understand how the sprinkler works, the pupils can do some research on the internet, try it themselves and describe the way of the water in their own words. For simplification they might reduce the number of dimensions and consider the initial speed and the angle of the water jet as constant.
2. To mathematise this model, the pupils might see that the water follows a parabola. They could measure or estimate the maximum height and length of the water jet. This data can also be given to them. However, they should be able to develop a quadratic equation that describes the water jet. That means, that they must adjust certain parameters. Finally, they need to ask questions like: What is the maximum height/width of the water jet? The pupils must find solution approaches like $f(x) = 0$.
3. To find mathematical results, they need to solve quadratic equations with or without the calculator or find out the maximum or the intersection point with the x-axis.
4. To get reals results, the solutions must be interpreted, answers to the questions must be given in a text and numbers must relate to units.
5. The last step is to look at the real situation again. Do the results match with the measurements? Is the model good enough? What can be optimised? Are there other questions that can be asked? Can we reduce the simplifications that were made in the beginning?

This example shows, how many possibilities can be given to a mathematics lesson by a simple situation from daily live. Single steps can be trained or be connected with others. The amount of information that is given, can be adjusted to bear in mind the individual skills of the pupils. Some pupils can work faster and go further in their investigations. All pupils can explore and see, that mathematics is connected to their daily lives. By using the modelling circle, it is possible to teach and train all key competences that are stated in the syllabus.