European Schools

Office of the Secretary-General

Pedagogical development Unit

Ref.: 2011-01-D-42-en-2

Orig.: FR

S7ma

MATHEMATICS SYLLABUS SECONDARY 7th YEAR

Further level 3 period/week course

Approved by the Joint Teaching Committee on 9, 10 and 11 February 2011 in Brussels

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Entry into force in September 2011

I. COMPULSORY TOPICS

THEME A: TRIGONOMETRIC FUNCTIONS (CIRCULAR AND HYPOBOLIC)

Topic	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Trigonometric Identities	*Pupils must be able to and/or understand* ▪ recall and/or derive the following formulae using complex numbers : ○ $\cos^2 a+\sin^2 a=1$ ○ $1+\tan^2 a=\dfrac{1}{\cos^2 a}$ ○ compound angle formulae : ▪ $\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$ ▪ $\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$ ▪ $\tan(a\pm b)=\dfrac{\tan a\pm\tan b}{1\mp\tan a\tan b}$ ○ double angle formulae : ▪ $\cos(2a)=\cos^2 a-\sin^2 a$ $=2\cos^2 a-1$ $=1-2\sin^2 a$ ▪ $\sin(2a)=2\sin a\cos a$ ▪ $\tan(2a)=\dfrac{2\tan a}{1-\tan^2 a}$ ○ t - formulae : ▪ $\cos a=\dfrac{1-t^2}{1+t^2}$ ▪ $\sin a=\dfrac{2t}{1+t^2}$ with $t=\tan\dfrac{a}{2}$ ▪ $\tan a=\dfrac{2t}{1-t^2}$	*Pupils must be able to and/or understand* ▪ use technological support to prove, check and understand the formulae given

Topic	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
	○ sum - product formulae : ▪ $\cos p + \cos q = = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$ ▪ $\cos p - \cos q = = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$ ▪ $\sin p \pm \sin q = = 2\sin\left(\frac{p\pm q}{2}\right)\cos\left(\frac{p\mp q}{2}\right)$ ○ product – sum formulae : ▪ $\cos a\cos b = \frac{1}{2}\left[\cos(a+b)+\cos(a-b)\right]$ ▪ $\sin a\cos b = \frac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$ ▪ $\sin a\sin b = \frac{1}{2}\left[\cos(a-b)-\cos(a+b)\right]$ ▪ $\cos^2 a = \frac{1+\cos(2a)}{2}$ ▪ $\sin^2 a = \frac{1-\cos(2a)}{2}$ ▪ solve an equation using these trigonometric formulae ▪ linearise an expression involving trigonometric functions ▪ express $\cos(nx)$ and $\sin(nx)$,as a power of $\cos x$, and $\sin x$	

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Inverse Trigonometric Functions	*Pupils must be able to and/or understand* ▪ given the inverse trigonometric functions, Arccos, Arcsin and Arctan: ○ give their definitions ○ give the domain for which the function is defined ○ give the limits of the functions ○ determine whether they are odd or even ○ give the domain for which they are differentiable ○ give their derivatives ○ determine where they are increasing/decreasing ▪ integrate using these functions ▪ explore (families of) functions involving trigonometric functions	*Pupils must be able to and/or understand* ▪ use the techniques covered in the 5 hour course to study these functions
Hyperbolic Functions Definitions and Formulae	*Pupils must be able to and/or understand* ▪ give the definitions of cosh, sinh and tanh ▪ derive the following formulae : ○ $\cosh a + \sinh a = e^{a}$ und $\cosh a - \sinh a = e^{-a}$ ○ $\cosh^2 a - \sinh^2 a = 1$ ○ $1 - \tanh^2 a = \frac{1}{\cosh^2 a}$ ○ addition formulae : ▪ $\cosh(a \pm b) = \cosh a \cosh b \pm \sinh a \sinh b$ ▪ $\sinh(a \pm b) = \sinh a \cosh b \pm \cosh a \sinh b$ ▪ $\tanh(a \pm b) = \frac{\tanh a \pm \tanh b}{1 \pm \tanh a \tanh b}$	*Pupils must be able to and/or understand* ▪ use technological support to prove, check and understand the formulae given

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	○ double angle formulae : ▪ $\cosh(2a) = \cosh^2 a + \sinh^2 a$ $= 2\cosh^2 a - 1 = 2\sinh^2 a + 1$ ▪ $\sinh(2a) = 2\sinh a \cosh a$ ▪ $\tanh(2a) = \frac{2\tanh a}{1+\tanh^2 a}$ ○ t- formulae : ▪ $\cosh a = \frac{1+t^2}{1-t^2}$ ▪ $\sinh a = \frac{2t}{1-t^2}$ with $t = \tanh\frac{a}{2}$ ▪ $\tanh a = \frac{2t}{1+t^2}$ ○ sum - product formulae: ▪ $\cosh p + \cosh q = 2\cosh\left(\frac{p+q}{2}\right)\cosh\left(\frac{p-q}{2}\right)$ ▪ $\cosh p - \cosh q = 2\sinh\left(\frac{p+q}{2}\right)\sinh\left(\frac{p-q}{2}\right)$ ▪ $\sinh p + \sinh q = 2\sinh\left(\frac{p+q}{2}\right)\cosh\left(\frac{p-q}{2}\right)$ ▪ $\sinh p - \sinh q = 2\cosh\left(\frac{p+q}{2}\right)\sinh\left(\frac{p-q}{2}\right)$ ○ product – sum formulae: ▪ $\cosh a \cosh b = \frac{1}{2}\left[\cosh(a+b) + \cosh(a-b)\right]$ ▪ $\sinh a \cosh b = \frac{1}{2}\left[\sinh(a+b) + \sinh(a-b)\right]$	

Topic	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
	▪ $\sinh a \sinh b = \frac{1}{2}\left[\cosh(a+b) - \cosh(a-b)\right]$ ▪ $\cosh^2 a = \frac{\cosh(2a)+1}{2}$ ▪ $\sinh^2 a = \frac{\cosh(2a)-1}{2}$ ▪ linearise an expression involving hyperbolic functions ▪ express $\sinh(nx)$ and $\cosh(nx)$, as powers of $\sinh(nx)$ and $\cosh(nx)$	
Hyperbolic Functions	*Pupils must be able to and/or understand* ▪ given the hyperbolic functions cosh, sinh, and tanh : o give the domain for which the the function is defined o give the limits of the functions o determine whether they are odd or even o give the domain for which they are differentiable o give their derivatives o determine they are increasing/decreasing ▪ integrate using these functions ▪ explore (families of) functions involving hyperbolic functions	*Pupils must be able to and/or understand* ▪ use the techniques studied in the 5 hour syllabus to study these functions

THEME B: LIMITS AND POWER SERIES

Topic	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Expansions: Classical Theorems and prerequisites.	*Pupils must be able to and/or understand* ▪ quote and apply theorems: ○ Rolle's Theorem ○ mean value theorem (Lagrange) ○ the inequality of the mean value theorem (and its corollaries) ▪ define a contracting function ▪ find a fixed point of a differentiable and contracting function ▪ define the *n*-th derivative of a function ▪ define a function of class $C(n)$ ▪ show the Taylor Expansion of order *n* in the case of a function of class $C(n)$, the polynomials of degree less than or equal to *n* ▪ identify the Taylor Expansion and MacLaurin Expansion of order *n* in the case of a function of class $C(n+1)$ ▪ use the MacLaurin Expansion to give an approximate value of a function at a value and increase the error	*Pupils must be able to and/or understand* ▪ determine the value or values for which Rolle's Theorem and mean value theorem are valid ▪ represent a sequence tending to a fixed point of a contracting function ▪ calculate the *n*-th derivative of a function of class $C(n)$ ▪ compute the Taylor or MacLaurin Expansion around a real number ▪ graph the Taylor Expansion of order *n* of a given function ▪ represent a family of expansions of a function ▪ use a Taylor Expansion of order *n* to give an approximate value of a function at a value
Expansions	*Pupils must be able to and/or understand* ▪ determine the Taylor Expansion of order n around zero of a function ▪ that the Taylor Expansion of order n, when it exists, is unique ▪ determine the expansions of order n around	*Pupils must be able to and/or understand* ▪ to determine the Taylor Expansion of order n of a function around any real number

Topic	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
	zero of the following functions: ○ $x \mapsto \frac{1}{1 \pm x}$ ○ $x \mapsto \ln(1 \pm x)$ ○ $x \mapsto (1+x)^n, n \in \left\{\frac{1}{2};2;3;...\right\}$ ○ $x \mapsto e^x$ ○ $x \mapsto \cos x$ ○ $x \mapsto \sin x$ ▪ find the expansions around zero for composite functions obtained the list above by sum, product, quotient, composition, integration, derivation ▪ find the expansion of the functions listed above around any non-zero number	Note: The emphasis of the use of technological support is on interpretation and application. ▪ Check the results found without a tool for simple cases. ▪ Use the tool for standard calculations to enable more time to be spent on interpretation and application.
Applications of expansions	*Pupils must be able to and/or understand* ▪ use expansions to: ○ calculate limits ○ find the approximation of a function around a point with a polynomial of degree equal to or greater than 2 ○ determine the relative position of the graph of a function with respect to one of its tangents ○ study the asymptotic behaviour of a function using a generalized Taylor expansion (e.g. parabolic branches)	*Pupils must be able to and/or understand* ▪ use the technological support wisely to apply the methods they have been taught

II. OPTIONAL TOPICS

IMPORTANT NOTE:

Unlike the preceding compulsory part of the program the description of each optional topic gives only a general overview of the content. Small adjustments in the content, linked to specific programs or requirements of national universities in different countries of the European Union remain possible. It is up to the teacher to make the necessary changes.

However, for the sake of readability and comparability of this part of the program, teachers in charge of this course must keep an accurate record of the adjustments made to the chosen options. This record will accompany the oral exam questions forwarded to the inspector responsible for mathematics in the European Schools. This will ensure that all such information (statement of the subject matter and the oral exam) is available to external examiners appointed for the oral tests.

1. Topological Ideas

- intuitive topology - Points, arcs, surfaces
- passage from intuitive topology to structured topology
- topological spaces - Models: topology of discs, of parallelepipeds, of spheres
- Hausdorf space
- homeomorphisms

2. Differential equations

Solutions of equations of the types:

- separable variables
- $y' + y \cdot f(x) = g(x)$

  $\rho_2 \cdot y'' + \rho_1 \cdot y' + \rho_0 = f(x)$

  where $f(x)$ is either 0 or a polynomial function or an exponential function or a trigonometric function

3. Further integration

- calculations of integrals of the type $\int \frac{P(x)}{Q(x)}dx$ where P and Q are polynomial functions (we restrict ourselves to cases where the roots are repeated real , distinct real or two complex conjugate)
- calculations of integrals based on the primitives that arise from the study of cyclis and hperbolic functions
- calculations of integrals using substitutions of the type: $t = \cos x$, $t = \sin x$, $t = \tan x$, $t = \tan\frac{x}{2}$, $t = \mathrm{ch}x$, $t = \sinh x$
- calculations of integrals by recurrence formulae, for example integrals of the type $\int_0^{\pi/2} \sin^m x \cos^n x\, dx$, $\int_0^{+\infty} x^n e^{-x} dx$, $\int_0^{\pi/2} x^n \sin x dx$, $\int_1^{e} (\ln x)^n dx$,…
- Wallis' Formula

4. Applications of integration

- calculation of the root mean square value of a function of time
- finding the position of the centroid of a plane area, symmetric solid or shell
- calculation of Moment of Inertia of a plane area, symmetric solid or shell
- length of a curve and area of a surface of revolution
- polar integration

5. Partial Differentiation

- functions of two variables - First order partial derivatives
- geometrical interpretations - Higher order derivatives
- Euler's first theorem for homogeneous functions
- differentials
- differentiation of composite functions
- directional derivatives - maxima, minima, saddle points

6. Series

- definition, general term of a series, n-th partial sum (limited to series of real numbers)
- series of positive terms, series of negative terms, alternating series, upper and lower bounds of a sequence
- convergence and divergence of a sequence
- a necessary condition of convergence of a series (if the series $\sum u_n$ converges, then the sequence (u_{tn}) converges to zero, the converse is false)
- some classic number series:
  - the geometric series $\sum a^n$ with $a \in \mathbb{R}$
  - is the harmonic series $\sum \frac{1}{n}$
  - the alternating harmonic series $\sum (-1)^n \frac{1}{n}$
  - the Riemann-series $\sum \frac{1}{n^\alpha}$ with $\alpha \in \mathbb{R}$
- convergence criteria for numerical series:
  - convergence criteria for geometric series
  - comparison criteria (you may limited this to series of positive terms)
  - convergence criterion for alternate series (Leibniz rule): if (u_n) is an alternating sequence, such that $(|u_n|)$ a decreasing sequence and $\lim\limits_{n \to +\infty} u_n = 0$ then the series $\sum u_n$ converges
  - convergence criterion for Riemann-series: if $\alpha > 1$ then a series with the general term $\frac{1}{n^\alpha}$ converges and if $\alpha \leq 1$ it diverges
  - d'Alembert's converge criteria : if $\lim\limits_{n \to \infty} \left| \frac{u_{n+1}}{u_n} \right| = l$ then the series with a general term (u_n) converges if $l < 1$, diverges if $l > 1$ and is indeterminable if $l = 1$
- absolutely convergent series and semi-convergent series

7. Plane Intersections of surfaces

- functions of two variables
- intersection of a surface by a plane
- intersection of a cylinders
- intersection of a cones
- the equation of an intersecting surface $z = x^2 + y^2$ et $z = xy$

8. Correlation and Regression

- method of least squares
- general methods
- covariance
- rank correlation - Spearman's coefficient and Kendall's Coefficient

9. Confidence Intervals, Hypothesis Tests and Chi-squared test.

- unbiased estimates of the population mean μ and variance σ^2
- distribution of the sample mean $\bar{X}$, when X is normally and non-normally distributed (The Central Limit Theorem)
- confidence intervals for the mean μ, of a normal population with known variance and unknown variance (large sample); of a non –normal distribution with known and unknown variance (large sample)
- hypothesis testing for the parameter p of a binomial distribution (small sample) and for the mean λ of a Poisson distribution
- hypothesis testing for the mean μ, of a normal population with known variance and unknown variance (large sample); of a non –normal distribution with known and unknown variance (large sample)
- the χ^2 significance test for independence and χ^2 goodness-of-fit test for the binomial, Poisson and Normal distributions

10. Barycentric calculus and affine (linear) geometry

- Barycentric calculus
- study of the functions $M \to \sum_1^n \alpha_i \overrightarrow{MA_i}$ and $M \to \sum_1^n \alpha_i \left\|\overrightarrow{MA_i}\right\|^2$
- affine functions
- particular cases:
  - isometrics
  - translations
  - enlargements
  - symmetries
  - orthogonal or not affinities

11. Conics

- definition of directrix, eccentricity, equation in a Cartesian orthonormal, reduced equation, parabola,
- establishment of a conic from an equation of the form $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$

12. Vector isometries in V^3

- isometry and the associated matrix
- the orthogonal group: norm, scalar product and bases
- eigenvalues and eigenvectors
- composition of vector isometrics
- classification of vector isometrics

13. Descriptive geometry

- point, line, plane
- changing the plane of projection
- intersections of lines and planes
- orthogonality of lines and planes
- geometrical problems in space

14. Classical geometry

- study of configurations, parallelism, orthogonality
- construction Problems
- minimal path problems
- use of transformations
- changing axes translation, rotation of orthonormal axes

15. Isometries of the affine Euclidean space E_3

- liaison between vector isometries and affine isometries
- study and classification of isometries

16. Polynomials

- vector space and ring of polynomials
- division using decreasing powers: uniqueness, quotient and remainder
- H.C.F. of two polynomials
- zeros of polynomials
- polynomials in several variables

17. Linear Transformations

- homomorphisms, endomorphisms and isomorphisms of a vector space
- image and kernel of a homomorphism
- matrix of a homomorphism, rank of a matrix, properties
- determinants and their properties
- solution of systems of linear equations

18. Linear and multilinear forms

- dual space of a vector space
- covectors
- dual basis
- linear forms and change of basis
- vector lines and planes in 3-dim space
- multilinear forms
- symmetric multilinear forms, alternate forms: determinants in 2-dim and 3-dim space
- determinant of an endomorphism
- effect of a change of basis
- linear independence

19. Arithmetic

- properties of natural
- euclidean divisibility, primes
- GCD and LCM of integers, Euclid's algorithm
- Bachet-Bezout Theorem: a and b are coprime if and only if there exist two integers u and v such that $au + bv = 1$
- Gauss theorem: if a and b are coprime and if a divides bc, then a divides c
- Euclid's theorem: if a prime p divides a product ab, then p divides a or p divides b
- congruences modulo *n*
  - Fermat's Little Theorem: For p prime and a and p prime, we have: $a^{p-1} \equiv 1 \bmod p$
  - Wilson's Theorem: For p prime, then: $(p-1)! \equiv -1 \bmod p$
  - solving a system of congruences $\begin{cases} x \equiv a \bmod u \\ x \equiv b \bmod v \end{cases}$
  - application to Cryptography and encryption

20. Vector functions

- functions from $\mathbb{R}$ to $\mathbb{R}^2$ (or $\mathbb{C}$) or to $\mathbb{R}^3$
- differential coefficient of a vector function
- differentiation of the product of a real function by a vector function
- differentiation of a scalar product
- differentiation of a vector product
- construction of plane curves

21. Dynamics of a point in the plane

- system of reference, movement of a point
- trajectory, velocity vector, acceleration vector
- composition of velocities, of accelerations

22. Special relativity (in 2 dimensions)

- kinematic diagrams - Time invariance: Galilean group of matrices $\begin{pmatrix} 1 & 0 \\ u & 1 \end{pmatrix}$
- velocity addition $w = u + v$
- isomorphisms of (G, x) and of $(R, +)$
- natural units $(c = 1)$
- velocity of light invariance: Lorentz group of matrices $\beta\begin{pmatrix} 1 & u \\ u & 1 \end{pmatrix}$ with $\beta = \frac{1}{\sqrt{1-u^2}}$
- velocity addition $w = \frac{u+v}{1+uv}$
- conditions $\|\vec{u}\| < 1$ and $\|\vec{w}\| > 1$
- Tachyons - contraction, dilatation, doppler effects

23. Non-linear systems

- sensitivity to initial conditions
- Feigenbaum-curve and the Mandelbrot set
- attractors
- Newton-iterations

24. Graph Theory

- definition
- types of graphs
- adjacent matrices
- Eulerian Graphs
- Hamiltonian Path
- spanning trees

25. Simplex algorithm

- formulation of a linear programming problem: objective function, constraints
- artificial variables: inflows or outflows variables
- matrix formulation of a problem
- background variables and non-basic variables
- solutions eligible basic primal solution
- simplex algorithm: pivot method
- optimal solution
- special Cases
- method of two phases

26. Mechanics

- acceleration
- Newton’s laws
- vectors
- using Newton’s laws
- moments
- impulse and momentum
- coefficient of friction
- projectiles
- work, energy and power
- Newton’s law of restitution
- centre of mass
- toppling
- elastic strings
- simple harmonic motion
- motion in a circle
- motion in a vertical circle

27. Algorithms and Programming

- basic algorithms (see chapter on numerical analysis of the advanced course of the sixth year)
- local and general variables
- definition of function and program
- management of inputs and outputs
- control statements:
  - conditional statements ("if ... then ..." "repeat ... until ... " "... as to ... ")
  - instructions in loops ("for ... ranging from ... to ... ")
- compiling a program
- applications in practical situations (analysis, numerical analysis, probability, statistics, geometry, ...)

28. Representation of numbers and binary arithmetic

- binary representation of positive integers
  - binary addition
  - binary multiplication
- hexadecimal, octal, bits, bytes and words
  - conversions
- representation of positive and negative numbers
  - sign-magnitude method
  - signed binary integers (2's complement)
  - positive binary fractions
  - signed binary fractions
  - fixed point number representation (2's complement)
  - floating point number representation (2's complement)
  - range of floating point representation
  - normalisation of floating point numbers
  - rounding modes
- floating point arithmetic operations
  - addition and subtraction
  - multiplication and division
- accuracy problems
  - machine precision
  - minimizing the effect of accuracy problems