European Schools

Office of the Secretary-General

Pedagogical development Unit

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S7P5 MATHEMATICS SYLLABUS SECONDARY 7th YEAR

Standard level 5 period/week course

Approved by the Joint Teaching Committee on 9, 10 and 11 February 2011 in Brussels

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Entry into force in September 2011

ALGEBRA (for guidance: 15 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Complex Numbers	*In addition to the concepts met in the 6th year pupils must be able to and/or understand:* ▪ represent a complex number geometrically (Argand diagram) ▪ determine the modulus and argument of a complex number and its inverse ▪ determine the magnitude and argument of the product and quotient of two complex numbers ▪ the three different forms of a complex number: ○ $x+iy$ ○ $r\cdot(\cos\theta+i\cdot\sin\theta)$ ○ $r\cdot e^{i\theta}$ convert from one form to another for simple cases. ▪ determine the nth root $(n\in\mathbb{N}, n\neq 0)$ of a complex number given in both forms $z=r\cdot(\cos\theta+i\cdot\sin\theta)$ and $z=r\cdot e^{i\theta}$ ▪ solve equations of the type $z^n=a$ $(a\in\mathbb{Z},\ n\in\{1,2,3\})$ and represent the solutions graphically *The argument* θ *,of a complex number, in all calculations without a technological tool is limited to the following:* $k\cdot\frac{\pi}{6}$ or $k\cdot\frac{\pi}{4}$ *where* $k\in\mathbb{Z}$	*Pupils must be able to and/or understand:* ▪ check calculations and solutions using CAS (computer algebra system) in the technological tool ▪ write a complex number in each of the three forms: ○ $x+iy$ ○ $r\cdot(\cos\theta+i\cdot\sin\theta)$ ○ $r\cdot e^{i\theta}$ ▪ solve, step by step, equations of the form $z^n=a$ $(a\in\mathbb{C},\ n\in\mathbb{N}, n\neq 0)$

ANALYSIS (for guidance: 55 periods)

The technological tool in sequences can be used to not only confirm what has been studied without a calculator but also to mirror, step by step, this process for a sequence not limited to sequences from the basic set. It may also be appropriate at times to use the tool to immediately give a solution.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Sequences (10 periods)	*In addition to the concepts met in the 6th year pupils must be able to and/or understand:* ▪ observe the behaviour of a sequence graphically as the relation (n,u_n) (time plot) ▪ determine whether a sequence is increasing or decreasing ○ Given the explicit nth term then let $u_n = f(n)$ and study f' ○ Given a recurrence relation u_n then let $u_n = f(u_{n-1})$ and study the sign of $u_{n+1} - u_n$ ▪ find M an upper and m a lower bound of a sequence: $u_n \leq M$ and $u_n \geq m$ ▪ apply the theorem of convergence/divergence : ○ an increasing sequence bounded by M converges ○ a decreasing sequence bounded by m converges ▪ for the basic set of recurrence relations in the form $u_n = f(u_{n-1}),\ n \in \mathbb{N}$ where ○ $f: x \mapsto ax+b$ ○ $f: x \mapsto \sqrt{ax+b}$ ○ $f: x \mapsto \dfrac{ax+b}{cx+d},\ c \neq 0$	*Pupils must be able to and/or understand:* ▪ enter a sequence both explicit and recurrence relation in an appropriate application (spreadsheet, CAS, graph etc.) ▪ given the explicit nth term or a recurrence relation calculate a particular term ▪ given the first terms of a sequence find the explicit nth term and or the recurrence relation where appropriate ▪ investigate the behaviour of a sequence graphically as the relation (n,u_n) (time plot) ▪ create a web diagram of a sequence of the form $u_n = f(u_{n-1}),\ n \in \mathbb{N}$ and find from the diagram possible limits ▪ investigate the behaviour of a sequence by manipulating the initial term ▪ find by solving the equation $f(L) = L$, the limit L of a convergent sequence

	graphically represent such sequences in a web diagram; using this diagram to observe whether a sequence is converging and to what limit ▪ Calculate a limit of sequence from the basic set of recurrence relations for simple cases	

The technological tool in exponentials and logarithms can be used to not only confirm what has been studied without a calculator but also to mirror, step by step, the same process. It may also be appropriate at times to use the tool to immediately give a solution.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Exponential and Logarithms	*In addition to the concepts met in the 6th year pupils must be able to and/or understand:* ▪ the properties of indices and logarithms : $a^m \cdot a^n = a^{m+n}$ $\log_a(u \cdot v) = \log_a u + \log_a v$ $a^{-m} = \frac{1}{a^m}$ $\log_a\left(\frac{1}{u}\right) = -\log_a u$ $(a^m)^n = a^{m \cdot n}$ $\log_a(u^n) = n \cdot \log_a u$ where $a,b,u,v \in \mathbb{R}^+ \setminus \{0,1\}$, $m,n \in \mathbb{R}$ ▪ the meaning of the number e	*Pupils must be able to and/or understand:* ▪ verify the properties of logarithms and powers ▪ solve equations and inequalities involving logarithms and/or exponentials, knowing when to reject an invalid solution

Unless further specified, students must be able to apply, without a technological tool, the concepts referred to in the second column 'knowledge and skills', for the following basic functions where $a, b, c \in \mathbb{R},\ \lambda \in \mathbb{Z}$:

- polynomials functions $P(x)$ of degree ≤ 3
- $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomial functions of degree ≤ 2
- $a+\lambda\sqrt{bx+c};\sqrt{ax^2+bx+c}$
- $a+\lambda\cos(bx+c);\ a+\lambda\sin(bx+c);\ \tan x$
- $c+\lambda\ln(ax+b);\ \lambda x^{\alpha}\ln x$ for $\alpha \in \{-2,-1,0,1,2\}$
- $\lambda e^{ax+b};\ ce^{2x}+de^{x}+f;\ c\cdot\left(e^{ax}+e^{-ax}\right);\ e^{ax}\left(ax^2+bx+c\right)$

The technological tool in following analysis can be used to not only confirm what has been studied without a calculator but also to mirror, step by step, this process for a function which is not limited to only the basic ones. It may also be appropriate at times to use the tool to immediately give a solution.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Study of Real Functions of One Real Variable	*In addition to the concepts met in the 6th year pupils must be able to and/or understand:* ▪ the concept of an inverse function ▪ study the natural logarithmic and the exponential function of base *e* ▪ examine the following characteristics for all the basic functions given above: ○ domain ○ intersection with the coordinate axes ○ limits ○ asymptotes ○ derivative and how it may vary ○ tangent at a point ○ extrema	*Pupils must be able to and/or understand:* ▪ draw the graph of a function ▪ choose appropriate units and a window so that the essential characteristics of a graph can be viewed and studied ▪ use the technological tool to perform, step by step, the calculations needed for examining the characteristics referred to in the second column ▪ manipulate by grabbing and moving the graphs of natural logarithms and the exponential to be able to observe the essential characteristics ▪ use sliders to investigate a family of functions with one or more parameters

<table>
<tr><td></td><td>
curvature and points of inflexion
sketch the graph
</td><td>

use the calculation tool (cas) to determine the inverse of a function
graph f and f^{-1} to observe the symmetry of the curves
study the inverse trigonometric functions $\arcsin x,\ \arccos x,\ \arctan x$
study the functions $x \mapsto \log_a x$ and $x \mapsto a^x$ where $a \in \mathbb{R}^+ \setminus \{0,1\}$

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<tr><td>Integration</td><td>
Pupils must be able to and/or understand:

understand the concept of a primitive
determine the primitives of the following functions: $x^k \left(k \in \mathbb{Q} \setminus \{-1\}\right),\ \frac{1}{x},\ \sin x,\ \cos x,\ e^x$
the concept of an integral on a closed interval $[a,b],\ a,b \in \mathbb{R}$
the concept of an improper integral $\int_a^{\infty} f(x)dx,\ \int_{-\infty}^{b} f(x)dx$ and $\int_{-\infty}^{\infty} f(x)dx$ where $a,b \in \mathbb{R}$ (calculate these for simple cases)
monotonicity properties & mean value of a function

$a \le b$ and $f(x) \ge 0$ on $[a,b]$ then $\int_a^b f(x)dx \ge 0$
$a \le b$ and $f(x) \le g(x)$ on $[a,b]$ then $\int_a^b f(x)dx \le \int_a^b g(x)dx$
$a \le b$ and $m \le f(x) \le M$ on $[a,b]$ then

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Pupils must be able to and/or understand:

solve problems with or without parameters involving
primitives
Integrals
calculation of areas
calculations of volumes
calculations of length of curves

Study not only pure examples but also applications from the fields of physics, biology, economics and others
</td></tr>
</table>

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

- the following properties of integrals:
  - $\int_a^a f(x)dx = 0$
  - $\int_b^a f(x)dx = -\int_a^b f(x)dx$
  - $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$
  - $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

    and $\int_a^b [\lambda f(x)]dx = \lambda \int_a^b f(x)dx$

    (Linearity of integrals)
- calculate the integrals by direct integration for the functions:

  $x^k \left(k \in \mathbb{Q} \setminus \{-1\}\right),\ \frac{1}{x},\ \sin x,\ \cos x,\ e^x$
- for the following functions or functions that can be reduced to them following a simple transformation (including substitution)
  - Polynomial function $P(x)$ of degree ≤ 3
  - $\frac{ax+b}{cx+d}$
  - $a + \lambda\sqrt{bx+c}$
  - $a + \lambda\cos(bx+c);\ a + \lambda\sin(bx+c);\ \tan x$
  - $(ax+b)\cdot\sin(cx),\ (ax+b)\cdot\cos(cx)$
  - $c + \lambda\ln(ax+b);\ \lambda x^{\alpha}\ln x$

	○ λe^{ax+b}; $ce^{2x} + de^{x} + f$; $c \cdot \left(e^{ax} + e^{-ax}\right)$; $e^{ax}\left(ax^{2} + bx + c\right)$ for $a, b, c \in \mathbb{R}$, $\lambda \in \mathbb{Z}$ und $\alpha \in \{-2, -1, 0, 1, 2\}$ calculate the integrals using: ○ integration by parts ○ Integration by substitution ▪ an integral interpreted graphically as an area ▪ application of integration to calculate ○ areas in the plane ○ volumes of revolution about the x-axis *(In this part, the aim is not to evaluate the computational capabilities of a student but an understanding of integration.)*	

GEOMETRY (for guidance: 40 periods)

The technological tool in geometry can be used to not only confirm what has been studied without a calculator but also to mirror, step by step, problems which require harder calculations. It may also be appropriate at times to use the tool to immediately give a solution.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY

Analytical Geometry in $\mathbb{R}^3$	*All these concepts should be resolved without the aid of a calculator. However the aim without a calculator is not to assess the computational ability of a student but their understanding of spatial geometry.*
	
	The entire work on geometry applies only to orthonormal coordinate systems.
	
	In addition to the concepts met in the 6th year pupils must be able to and/or understand:
	
	▪ find the cartesian equation of a sphere
	▪ determine the relative position (points of intersection and / or equations where appropriate) of two of the following objects:
	  ○ point
	  ○ line
	  ○ plane
	  ○ sphere
	▪ determine the following orthogonal projections:
	  ○ point onto a plane
	  ○ point onto a line
	  ○ line onto a plane
	▪ find the centre and radius of the circle formed by a sphere with a plane and two intersecting spheres
	▪ calculate the following shortest distances between:
	  ○ two points missing
	  ○ a point and a plane
		The technological tool allows students to solve geometric problems numerically using sets of equations derived from problems with two or more objects (for example: three or more planes, two spheres and a plane etc.)
		
		Pupils must be able to and/or understand:
		
		▪ calculate the cross and dot product of two vectors

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
	○ two parallel planes ○ a line parallel to a plane ○ two lines ○ a point and a sphere ○ a line and a sphere ▪ calculate the acute angle if it exists made by: ○ two lines ○ two planes ○ a line and a plane ▪ find the equation of the line of a common perpendicular between two lines ▪ find the equation of the plane of symmetry of two points ▪ find the equation(s) of the plane(s) of symmetry of two parallel planes and two intersecting planes	

PROBABILITY AND STATISTICS (for guidance: 40 periods)

The technological tool in probability not only replaces paper versions of statistical tables but allows pupils the opportunity to explore and apply the concepts introduced in the column ‘knowledge and skills’. It may also be appropriate at times to use the tool to immediately give a solution.

A theoretical application or proof of the formulae contained in this section can never form part of a baccalaureate exam question. In the probability section the acquired knowledge and skills should only be applied in simple cases whenever technological support is not used.

<table>
<tr><th>TOPIC</th><th>KNOWLEDGE & SKILLS</th><th>USE OF TECHNOLOGY</th></tr>
<tr><td>Conditional Probability</td><td>In addition to the concepts met in the 6th year pupils must be able to and/or understand:
▪ that two independent events is a special case of conditional probability
▪ apply up to three independent events : $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$
▪ Identical repetitions of three Bernoulli trials
▪ know and apply, for $n = 2$:
o the law of total probability: $P(B) = P(A_1) \times P(B|A_1) + P(A_2) \times P(B|A_2)$
o Bayes’ Theorem: $P(A_k|B) = \frac{P(A_k) \times P(B|A_k)}{P(B)}$, $k \in \{1; 2\}$</td><td>Pupils must be able to and/or understand:
▪ apply up to n independent events
▪ apply the law of total probability, conditional probability and Bayes’ theorem for $n \leq 3$</td></tr>
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TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY

Discrete Distributions	*In addition to the concepts met in the 6th year pupils must be able to and/or understand:*
	
	▪ to understand the following concepts:
	  ○ discrete random variable
	  ○ probability distribution of a discrete random variable and the probability distribution function of a discrete random variable
	  ○ cumulative distribution function of a discrete random variable
	$F(x)=P(X\le x)=\sum_{x_i\le x}p(x_i)$
	  ○ expected value, variance and standard deviation of a discrete random variable
	$E(X)=\sum_{i=1}^{n}x_i\cdot P(X=x_i)$,
	$Var(X)=E(X^2)-E^2(X)$ where
	$E(X^2)=\sum_{i=1}^{n}x_i^2\cdot P(X=x_i)$
	▪ determine and apply the probability density function of a binomial random variable
	$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$
	▪ know and apply the formulas for the mean, variance and standard deviation of a random variable which is distributed binomially
	$\mu=np$ and $Var=\sigma^2=np(1-p)$
		All calculations using probability distributions will be done using a technological tool
		
		Pupils must be able to and/or understand:
		
		▪ calculate probabilities for a binomially distributed random variable step by step using
		$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$
		▪ find cumulative probabilities for a binomially distributed random variable
		▪ graphically represent probabilities
		▪ calculate the variance and standard deviation of a random variable that follows a binomial distribution
		
		(Students will be able to use a spreadsheet to develop the above ideas of a binomial variable step by step)
		
		▪ Apply and use the Poisson distribution for different sized intervals.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Continuous Distributions	*Pupils must be able to and/or understand:* ▪ the following concepts: ○ continuous random variable ○ probability density function (p.d.f) of a continuous random variable and its relation to integral calculus: $f \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$ ○ cumulative distribution function for a continuous random variable and its relation to integral calculus: $P(a \leq X \leq b) = \int_{a}^{b} f(x)dx$ and $F(k) = P(X \leq k) = \int_{-\infty}^{k} f(x)dx$, where $f(x)$ is the p.d.f. ○ mean (expected value), variance and standard deviation of a continuous random variable: $E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$ and $V(X) = E(X^2) - E^2(X)$, where $E(X^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)dx$ ▪ the concept of the normal distribution and the standardised variable : $z = \frac{x-\mu}{\sigma}$	*Pupils must be able to and/or understand:* ▪ use the normal distribution ▪ standardise any normal distributions $z = \frac{x-\mu}{\sigma}$

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Modelling	*Pupils must be able to and/or understand:* ▪ the conditions when a variable can be modelled by the following distributions: ○ discrete uniform ○ continuous uniform ○ binomial ○ Poisson ▪ question the suitability of a model by checking that the conditions of a distribution have been me	*Pupils must be able to and/or understand:* ▪ appropriately model situations by the following distributions ○ discrete uniform ○ continuous uniform ○ binomial ○ Poisson ○ Normal ▪ investigate data given in a table or a diagram to determine a corresponding normal distribution ▪ recognise the conditions for a Poisson approximation to the binomial $(n > 50 \text{ and } p < 0.1)$ ▪ recognise the conditions for a normal approximation to the binomial $(npq > 9)$ and use this approximation including the continuity correction $z = \frac{x \pm 0.5 - \mu}{\sigma}$.