



European Schools

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Pedagogical Development Unit

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# **S7P3** MATHEMATICS SYLLABUS SECONDARY 7th YEAR

## Elementary level 3 period/week course

Approved by the Joint Teaching Committee on 9, 10 and 11 February 2011 in Brussels

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Entry into force in September 2011

**ANALYSIS (for guidance: 40 periods)**

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
<p><b>Growth and Decay</b></p>	<p><i>The primary objective of this unit is to understand examples of growth and decay that follow logarithmic or exponential models. After defining the concepts, their application will be illustrated by the study of, for example, variations:</i></p> <ul style="list-style-type: none"> <li>▪ <i>of a population;</i></li> <li>▪ <i>of the temperature of an object;</i></li> <li>▪ <i>of the concentration of a substance in solution;</i></li> <li>▪ <i>of financial investments</i></li> </ul> <p><i>This list is not exhaustive.</i></p> <p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ solve simple equations of the form <math>a^x = b</math> (<math>a, b</math> natural numbers)</li> <li>▪ understand the relationship between the powers of a positive number, <math>a</math>, and logarithms with base <math>a</math></li> <li>▪ define the number <math>e</math>, and functions <math>x \mapsto e^x</math> and <math>x \mapsto \ln x</math></li> <li>▪ for the functions <math>x \mapsto k \cdot e^{ax+b}</math> and <math>x \mapsto k \cdot \ln(ax + b)</math>: <ul style="list-style-type: none"> <li>○ give the domain and range</li> <li>○ find the limits</li> <li>○ calculate the derivative function</li> <li>○ determine the intervals where the functions is increasing/decreasing</li> <li>○ find the values of <math>k, a</math> and <math>b</math> given the graph</li> </ul> </li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>• solve simple equations of the form <math>a^x = b</math> (<math>a, b</math> positive, real numbers)</li> <li>• plot and manipulate the graphs of functions <math>x \mapsto e^{ax+b}</math> and <math>x \mapsto \ln(ax + b)</math></li> <li>• plot the graphs of functions <math>x \mapsto k \cdot e^{ax+b}</math> and <math>x \mapsto k \cdot \ln(ax + b)</math></li> <li>• solve graphically and/or algebraically equations involving exponential and/or logarithmic functions</li> </ul>

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	<p>of the functions <math>x \mapsto k \cdot e^{ax+b}</math> and <math>x \mapsto k \cdot \ln(ax + b)</math></p> <ul style="list-style-type: none"> <li>▪ solve graphically and algebraically equations of the forms <math>e^{ax+b} = c</math> and <math>\ln(ax + b) = c</math> where <math>a</math>, <math>b</math> and <math>c</math> are real numbers</li> <li>▪ use all of the concepts and results defined above to study and interpret practical situations</li> </ul>	
<p><b>Using Integration in Problem Solving</b></p>	<p><i>This unit aims to introduce integral calculus as a tool for solving practical problems. Therefore emphasis will be put on:</i></p> <ul style="list-style-type: none"> <li>▪ <i>calculating arc lengths, areas of surfaces and volumes of solids of revolution;</i></li> <li>▪ <i>solving problems from physics, biology, economics, and other applications for which all relevant details will be provided</i></li> </ul> <p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ understand the method of rectangles to calculate an approximate value of an area bounded by the graph of a continuous positive function, the x-axis and two lines parallel to the y-axis; and that reducing the width of the rectangles improves this approximation</li> <li>▪ understand that the exact calculation of the area <math>A</math> is given by a definite integral denoted by</li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <p>Given a function determine:</p> <ul style="list-style-type: none"> <li>▪ the indefinite integral</li> <li>▪ find the primitive satisfying an initial condition</li> <li>▪ calculate the area of a surface, a volume of revolution or arc length of a curve, including: <ul style="list-style-type: none"> <li>○ the area bounded by the curves representing two functions</li> <li>○ the volume of a hollow solid</li> </ul> </li> </ul>

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	$A = F(b) - F(a) = \int_a^b f(x) dx$ <ul style="list-style-type: none"> <li>▪ use the following properties of definite integrals and interpret them graphically <ul style="list-style-type: none"> <li>○ <math>\int_a^a f(x) dx = 0</math></li> <li>○ <math>\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx</math></li> <li>○ <math>\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx</math></li> <li>○ <math>\int_a^b k \cdot f(x) dx = k \int_a^b f(x) dx, k \in \mathbb{R}</math></li> </ul> </li> <li>▪ for polynomials of degree less than or equal to 3, and functions of the types <math>x \mapsto k \cdot e^{ax+b}</math> and <math>x \mapsto \frac{k}{ax+b}</math>: <ul style="list-style-type: none"> <li>○ find the indefinite integral</li> <li>○ find the primitive satisfying an initial condition</li> <li>○ find a definite integral</li> <li>○ calculate the area bounded by the graph of the function (positive, negative for part or all), the x-axis and two vertical lines</li> </ul> </li> <li>▪ use all of the concepts and results defined above to study, interpret and solve problems set in practical situations</li> </ul>	<p><i>Remark:</i></p> <p>The formulae will always be given when required for:</p> <ul style="list-style-type: none"> <li>▪ the volume of revolution <math>\pi \int_a^b (f(x))^2 dx</math></li> <li>▪ the area bounded by the graphs of two functions <math>\int_a^b  f(x) - g(x)  dx</math></li> <li>▪ the length of the arc of a curve <math>\int_a^b \sqrt{1 + (f'(x))^2} dx</math></li> </ul>

**PROBABILITY (for guidance: 25 periods)**

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<b>Random Variables</b>	<p><i>As in 6th year, the concepts of this unit should be taught in a practical context. Theory and formal proof are not required. The mathematical skills must be mastered in a practical context. There are many varied applications from where examples and exercises can be given (data processing in economics, geography, physics, biology ...)</i></p>	
<b>Discrete Random Variables</b>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ explain the conditions under which a random variable follows a binomial distribution</li> <li>▪ that the binomial distribution can be used as an approximation when a small sample is taken by making successive draws from a large population</li> <li>▪ understand and interpret the concepts of expectation and standard deviation of a random variable <math>X</math>, in particular when <math>X</math> is distributed binomially (<math>B(n, p)</math>) and know that <math>E(X) = np</math> and <math>\sigma(X) = \sqrt{np(1-p)}</math></li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>• For a discrete random variable following a binomial distribution calculate               <ul style="list-style-type: none"> <li>○ <math>P(X = k)</math></li> <li>○ <math>P(X \leq k)</math></li> <li>○ <math>P(X \geq k)</math></li> <li>○ <math>P(k \leq X \leq k')</math></li> </ul> </li> </ul>

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<b>Continuous Random Variables</b>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ that in some cases there is a need for continuous random variables and the corresponding probabilities are calculated using integrals with graphical interpretation (no theoretical development is necessary)</li> <li>▪ many sets of data can be modelled by a normal distribution, characterized by its mean and standard deviation</li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ represent data as a dot plot and find by manipulating a graph of a normal distribution curve find an appropriate fit</li> <li>▪ for a random variable following a normal distribution calculate: <ul style="list-style-type: none"> <li>○ <math>P(X \leq b)</math></li> <li>○ <math>P(a \leq X)</math></li> <li>○ <math>P(a \leq X \leq b)</math></li> </ul> </li> <li>▪ Find <math>a</math>, given both <math>P(X \leq a) = \alpha</math> and the normal distribution of <math>X</math>.</li> </ul>
<b>Random Variables (discrete or continuous)</b>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ use all the concepts and results defined above, including the properties of probabilities seen in previous years (including conditional probability) to study and interpret practical situations</li> </ul>	

**STATISTICS (for guidance: 25 periods)**

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<p><b>Prerequisites for Statistics</b></p>	<p>Revision of the concepts studied in years 1-5 that were not included in the sixth year syllabus. This should be based on practical examples using technological support where possible and without any theoretical proofs.</p> <p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ understand and use linear functions</li> <li>▪ identify and interpret arithmetic mean, mode(s), median, upper and lower quartiles and interquartile range of data in one variable</li> <li>▪ understand how to use a spreadsheet</li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ calculate the mean, standard deviation, median, upper and lower quartiles and interquartile range of data in one variable</li> <li>▪ enter data into a spreadsheet</li> <li>▪ use a spreadsheet, including relations between various cells, to automate the calculations</li> </ul>
<p><b>Analysing and Representing data in one variable.</b></p>	<p><i>The emphasis of this unit is on interpretation. Only examples drawn from everyday life (e.g. wages in a business, production of a machine or a biological study, etc...) must be used.</i></p> <p><i>In examples and exercises without technological support, it is sufficient to use not more than 10 items of data.</i></p> <p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ calculate a weighted mean</li> <li>▪ use the properties of linearity of the mean, that is</li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ calculate a weighted mean</li> <li>▪ use box plots to represent one or more sets of data</li> </ul>

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	<p>to say:</p> <ul style="list-style-type: none"> <li>○ if <math>t_i = x_i + a</math>, then <math>\bar{t} = \bar{x} + a</math></li> <li>○ if <math>z_i = b \cdot y_i</math>, then <math>\bar{z} = b \cdot \bar{y}</math></li> </ul> <ul style="list-style-type: none"> <li>▪ use box plots to represent one or more sets of data in one variable</li> <li>▪ analyse and compare two sets of data given the mean, median, quartiles and extreme values</li> <li>▪ summarize a set of data using a pair of values (measures of central tendency and dispersion). That is either (mean and SD) or (median and IQR), and understand that the second pair is particularly useful if there are some extreme values</li> <li>▪ use all of the concepts and results given above to study and interpret practical situations</li> </ul>	<p>in one variable.</p>
<p><b>Bivariate Data</b></p>	<p><i>The main objective in this unit is to use interpolation and extrapolation to make predictions. There are many practical applications e.g. economics, populations, geography, sciences, etc. In examples and exercises without technological support it is sufficient to use not more than 10 pairs of data. The independent variable will be given by the teacher.</i></p> <p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ represent bivariate data in a scatter diagram and appreciate the usefulness of a possible relationship between the variables.</li> <li>▪ determine the coordinates of the mean point <math>(\bar{x}, \bar{y})</math></li> </ul>	<p><i>Pupils must be able to and/or understand:</i></p> <ul style="list-style-type: none"> <li>▪ enter bivariate data in a spreadsheet and draw a scatter diagram</li> <li>▪ manipulate the graphs of known functions to fit data represented by a scatter diagram</li> </ul>



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	<p>and mark it on a scatter diagram</p> <ul style="list-style-type: none"> <li>▪ organise in ascending order the data by the independent variable <math>x</math>, then split them by putting the first half in one group and the second in another. Find the mean point of each of these subset and connect the two points with a straight line (called line of Mayer) to create an estimate of the linear relationship between the data</li> <li>▪ understand the method of least squares when calculating the least square regression line (regression of <math>y</math> on <math>x</math>)</li> <li>▪ on a scatter diagram by using trial and error, find an appropriate regression model (linear, logarithmic or exponential), taking care to identify and eliminate outliers where necessary.</li> <li>▪ examine the suitability of a linear regression model by calculating and interpreting Pearsons' product moment correlation coefficient</li> <li>▪ appreciate that the line of Mayer and the line of regression of <math>y</math> on <math>x</math> passes through the mean point of the data on a scatter diagram</li> <li>▪ use a regression model to make interpolations, extrapolations and forecasts</li> <li>▪ use all of the concepts and results given above to practical applications</li> </ul>	<ul style="list-style-type: none"> <li>▪ calculate the mean point and add it to the scatter diagram</li> <li>▪ show graphically the idea that the linear regression model will best fit the data when the sum of the least squares is minimised. (<math>y</math> on <math>x</math>)</li> <li>▪ calculate Pearsons' product-moment correlation coefficient</li> <li>▪ find the equation of the line of Mayer and the line of linear regression of <math>y</math> on <math>x</math></li> <li>▪ draw the line of Mayer and the line of linear regression of <math>y</math> on <math>x</math></li> <li>▪ for given bivariate data which is either exponential or logarithmic apply a given (by the teacher) change of variable and observe from the scatter diagram that a linear regression model would be appropriate. From this conclude that the original data is exponential or logarithmic: <ul style="list-style-type: none"> <li>○ <math>(\ln y = a \cdot x + b) \Rightarrow (y = c \cdot d^x)</math></li> <li>○ <math>y = a \cdot \ln x + b</math></li> </ul> </li> <li>▪ find a linear, exponential or logarithmic regression equation for bivariate data by using the regression tools.</li> </ul>