European Schools

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Pedagogical Development Unit

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S5P6

MATHEMATICS SYLLABUS SECONDARY 5th YEAR

6 period/week course

APPROVED BY THE JOINT TEACHING COMMITTEE ON 9, 10 AND 11 FEBRUARY 2011 IN BRUSSELS

Entry into force in September 2011

ALGEBRA (for guidance: 80 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Absolute Value	*Pupils must be able to and/or understand :* ▪ use the rules concerning the sum, difference, product and quotient of absolute values ▪ solve equations and inequalities of the kind $\lvert ax+b\rvert+c\geq 0,\ \lvert ax+b\rvert=c$	*Pupils must be able to and/or understand :* ▪ solve equations and inequalities involving the absolute value ▪ use graphs to solve equations and inequalities
Powers and roots	*Pupils must be able to and/or understand :* ▪ express the n^{th} roots of a real number (and the powers of these roots) as powers with rational indices ▪ understand that finding the n^{th} root and raising to the n^{th} power are inverse operations ▪ understand that the rules of calculation for powers where the index is a whole number extends to powers where the index is a rational number ▪ use these properties in problems involving exponential growth and decay ▪ simplify algebraic expressions like: $3^{-2}\times 9^{2},\ \frac{16^{\frac{1}{2}}}{4},\ \frac{21a^2b}{7ab^2},\ 3a^{-\frac{1}{2}}b\times 18ab$ ▪ explore values of x in simple exponential equations such as $2^x=4^{2x+1}$ to find possible solutions	*Pupils must be able to and/or understand :* ▪ use the tool to calculate expressions and verify solutions ▪ use a spreadsheet and a scatter plot to solve problems involving exponential growth and decay ▪ solve and verify solutions of simple exponential equations such as $2^x=4^{2x+1}$

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Simultaneous Equations of the type: $\begin{cases} ax+by+cz=d \\ ex+fy+gz=h \\ ix+jy+kz=l \end{cases}$	*Pupils must be able to and/or understand :* ▪ solve these systems algebraically ▪ form and solve systems from simple worded problems ▪ form and solve a system of equations to find the coefficients of the function of a parabola passing through three given points for simple cases	*Pupils must be able to and/or understand :* ▪ solve these systems ▪ use the tool to fit a parabola given three points ▪ use the tool to verify the equation of the parabola
Polynomials	*Pupils must be able to and/or understand :* ▪ divide $P(x)$ by $Q(x)$ (where $Q(x)$ is of first or second degree only) ▪ understand and apply the remainder theorem ▪ factorise a polynomial (easy cases, degree ≤ 4) ▪ apply the identities below (and know them) ○ $a^3+b^3=(a+b)(a^2-ab+b^2)$ ○ $a^3-b^3=(a-b)(a^2+ab+b^2)$ ○ $(a\pm b)^3=a^3\pm 3a^2b+3ab^2\pm b^3$ ▪ find the zeros of polynomials ▪ find the signs of polynomials ▪ simplify, add, subtract, multiply and divide rational fractions ▪ study the variations in sign of a rational fraction $\frac{P(x)}{Q(x)}$, where the degree of $P(x)$ and $Q(x)$ is less or equal to 2	*Pupils must be able to and/or understand :* ▪ simplify expressions involving divisions of polynomials ▪ factorise a polynomial ▪ find the zeros of polynomials ▪ find the signs of polynomials ▪ simplify, add, subtract, multiply and divide rational fractions $\frac{P(x)}{Q(x)}$

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Quadratic equations and inequalities	*Pupils must be able to and/or understand :* ▪ understand and apply the relation between the coefficients of a quadratic equation and its solutions: $ax^2 + bx + c = 0$ which can be expressed as $(x-\alpha)(x-\beta) = 0$ where $\alpha + \beta = \frac{-b}{a}$ and $\alpha \cdot \beta = \frac{c}{a}$ ▪ solve quadratic inequalities ▪ solve equations which can be reduced to a quadratic equation (e.g. a biquadratic equation) [simple cases only] ▪ solve worded problems which lead to equations of the above types	*Pupils must be able to and/or understand :* ▪ solve quadratic inequalities algebraically and graphically ▪ solve equations which can be reduced to a quadratic equation
Real functions	*Pupils must be able to and/or understand :* ▪ use the idea of the domain of definition for the following functions ○ polynomial functions of order 3 or less ○ $f(x) = \sqrt{ax+b}$ ○ $f(x) = \sin x$ ○ $f(x) = \cos x$ ○ $f(x) = \tan x$ ○ $f(x) = \frac{ax+b}{cx+d}$ ▪ for functions of the type $y = \sqrt{ax+b}$ ○ draw and recognise their graphs ○ find the zero and, where appropriate, the y intercept	*Pupils must be able to and/or understand :* ▪ verify that graphs have been successfully drawn ▪ find the zeros and the y intercept graphically ▪ deduce from a graph the equations of the asymptotes of a hyperbola ▪ draw the graphs of a large variety of the real functions

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	▪ for functions of the type $y = \sin x$, $y = \cos x$ and $y = \tan x$ ○ draw and recognise their graphs ○ understand the idea of the period of these functions ▪ for rectangular hyperbolas of the type $y = \frac{a}{x+b}$ and $y = \frac{ax+b}{cx+d}$ ○ draw and recognise their graphs ○ determine that $\frac{ax+b}{cx+d} = A + \frac{B}{cx+d}$ ○ give the equation of the vertical and horizontal asymptotes ○ find the zero and the y intercept, when appropriate ○ know the behaviour of the graph at positive and negative infinity ○ know the behaviour of the graph close to the vertical asymptote ○ determine the centre of symmetry ▪ draw and recognise the graphs $y = \lvert g(x) \rvert$ for the following functions: ○ $g(x) = ax + b$ ○ $g(x) = ax^2 + bx + c$ ○ $g(x) = \frac{ax+b}{cx+d}$ ▪ find, where appropriate, for $y = \lvert g(x) \rvert$ ○ the equations of the asymptotes ○ the centre of symmetry	

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	○ the zero ○ the y intercept ▪ sketch the graphs of different piecewise functions composed of functions of the types ○ $g(x) = ax + b$ ○ $g(x) = ax^2 + bx + c$ ○ $g(x) = \frac{ax + b}{cx + d}$ e.g. $f(x) = \begin{cases} -3x - 2 & x \leq -1 \\ -x^2 + 2 & x > -1 \end{cases}$	

PROBABILTY and STATISTICS (for guidance: 52 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Elementary ideas in probability	*Pupils must be able to and/or understand :* ▪ determine the universal set Ω of possible outcomes of a random trial ▪ define an event *A* as sub-set of Ω (containing one or more elements) ▪ show the universal set in the form of a Venn diagram	
Algebra of events	*Pupils must be able to and/or understand :* ▪ express the events $A \cup B$ and $A \cap B$ in words and in set form ▪ know that, for exclusive events, $A \cap B = \varnothing$ ▪ find a complementary event $\overline{A}$	
Probability	*Pupils must be able to and/or understand :* ▪ understand for the probability of an event *A* that: $0 \le P(A) \le 1$ ▪ calculate the probability of an event *A* ▪ understand the idea of probability leading on from relative frequency	*Pupils must be able to and/or understand :* ▪ use the random number generator ▪ create by using a spreadsheet, the relative frequency of a virtual generated experiment and compare with the theoretical probabilities (e.g. dice simulation) ▪ calculate probability values as a fraction and as a decimal

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Combined events	*Pupils must be able to and/or understand :* ▪ use for independent events (sampling **with replacement**): ○ total possibility space diagrams ○ Venn diagrams ○ tree diagrams ▪ use for conditional events (sampling **without replacement**), tree diagrams (limit to up to three sets of branches) ▪ solve problems which require the following: $P(\bar{A}) = 1 - P(A)$ for complementary events $P(A \cap B) = 0$ for mutually exclusive events $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B)$ for mutually exclusive events $P(A \cup B) = 1$ for exhausitive events $P(A \cap B) = P(B) \cdot P(A \mid B)$ (use only in tree diagrams) $\left[P(A \cap B) = P(B) \cdot P_B(A)\right]$ $P(A \cap B) = P(A) \cdot P(B)$ for independent events	*Pupils must be able to and/or understand :* ▪ use the tool in probability problems

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Analysis of data	*Pupils must be able to and/or understand :* ▪ understand that the variance and the standard deviation are a measure of spread ▪ calculate, for a small size sample ($n \leq 6$), variance and standard deviation, using one of the following $\sigma^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n} = \frac{\sum x^2}{n} - \left(\bar{x}\right)^2$	*Pupils must be able to and/or understand :* ▪ calculate from frequency distributions or histograms the variance and the standard deviation ▪ calculate an estimate of the variance and the standard deviation for raw and grouped data $\sigma^2 = \frac{\sum\left(x-\bar{x}\right)^2}{n} = \frac{\sum x^2}{n} - \left(\bar{x}\right)^2$ $\sigma^2 = \frac{\sum a\left(x-\bar{x}\right)^2}{\sum a} = \frac{\sum ax^2}{\sum a} - \left(\bar{x}\right)^2$ ▪ calculate the variance and the standard deviation for raw and grouped data using a spreadsheet
Interpretation and comparison of data	*Pupils must be able to and/or understand :* ▪ compare and interpret distributions with respect to: ○ the means and variances/standard deviations ○ their given histograms	*Pupils must be able to and/or understand :* ▪ use a tool to calculate an estimate of the mean, variance and the standard deviation ▪ construct histograms for interpretation and comparison

GEOMETRY (for guidance: 60 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Oriented angles	*Pupils must be able to and/or understand :* ▪ define the unit circle ▪ define an oriented angle and represent it in the unit circle ▪ define radians in relation to the arc length of a sector ▪ convert radians to degrees and vice-versa ▪ estimate the size of an angle in radians and degrees	*Pupils must be able to and/or understand :* ▪ convert radians to degrees and vice versa ▪ use constructions and measurements to verify the estimates of an angle
Trigonometric Ratios	*Pupils must be able to and/or understand :* ▪ find the trigonometric ratios of an oriented angle and of angles associated with it in both degrees and radians ▪ know the geometric meaning of the trigonometric ratios ▪ state how the trigonometric ratios of a signed angle vary ▪ state the trigonometric ratios of certain special angles ▪ compare the trigonometric ratios of angle with those of its: ○ complementary angle: $\cos\theta = \sin\left(90^\circ - \theta\right)$ $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ ○ supplementary angle: $\sin\theta = \sin\left(180^\circ - \theta\right)$ $\sin\theta = \sin\left(\pi - \theta\right)$ ▪ know and apply formulae of the type ○ $\sin^2\theta + \cos^2\theta = 1$	*Pupils must be able to and/or understand :* ▪ use graphs of the trigonometric functions to understand that other angles will give the same trigonometric ratios as those of the standard angles ▪ show complementary and supplementary properties dynamically using sliders ▪ do calculations involving trigonometric formulae ▪ verify solutions to trigonometric equations ▪ solve trigonometric equations

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	○ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ▪ know and apply (only with numerical values of angles) formulae of the type ○ $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$ ○ $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$ ○ $\sin(2\alpha) = 2\sin\alpha\cos\alpha$ ○ $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$ ▪ solve, for $0 < \theta < 2\pi$ and $\theta \in \mathbb{R}$ ○ equations of the types: $\sin\theta = a$, $\cos\theta = a$ and $\tan\theta = a$ ○ simple equations involving these such as: $\cos\left(\theta + \frac{\pi}{6}\right) = \frac{1}{2}$ and $3\cos^2\theta - \sin\theta - 1 = 0$	
Triangles	*Pupils must be able to and/or understand :* ▪ state and prove the following formulae for any triangle ○ $a^2 = b^2 + c^2 - 2bc \cdot \cos\hat{A}$ ○ $\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = 2r$ ○ $\text{Area} = \frac{1}{2}bc \cdot \sin\hat{A}$ ▪ apply the formulae to determine angles and sides of triangles, including real life problems	*Pupils must be able to and/or understand :* ▪ verify these formulae using construction and measurements ▪ solve problems involving these formulae
Vectors in the plane	*Pupils must be able to and/or understand :*	

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	▪ recognise the following: ○ linearly dependent and independent vectors, basis, coordinate system ○ the dimension of a vector space ▪ define an orthonormal basis ▪ express a vector as a linear combination of two given vectors that form a basis ▪ show the bijection which exists between the set of vectors and the set of ordered pairs of real numbers	
Scalar product	*Pupils must be able to and/or understand :* ▪ define and calculate the scalar product of two vectors ▪ define the scalar product of a vector with itself ▪ define the magnitude of a vector ▪ define the orthogonality of two vectors ▪ list and use the properties of the scalar product ▪ express the scalar product of two vectors in terms of their magnitudes and the cosine of the angle between them ▪ use the scalar product to verify orthogonality ▪ express a scalar product in an orthonormal basis ▪ calculate the distance between two points ▪ use vector methods for geometric proofs for instance ○ $\frac{a}{\sin\hat{A}} = \frac{b}{\sin\hat{B}} = \frac{c}{\sin\hat{C}} = 2r$ ○ $\text{Area} = \frac{1}{2}bc \cdot \sin\hat{A}$	*Pupils must be able to and/or understand :* ▪ calculate the scalar product of two vectors ▪ show dynamically the properties of the scalar product using construction and measurements

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Lines in the plane	*Pupils must be able to and/or understand :* ▪ give the vector equation of a straight line ▪ find a parametric equation of a line ▪ find a cartesian equation of a line ▪ determine the relative position of two lines ▪ find parallel lines and recognise parallel lines from their equations ▪ determine the equation of a line parallel to a given line, passing through a given point ▪ find perpendicular lines and recognise perpendicular lines from their equations ▪ determine the equation of a line perpendicular to a given line, passing through a given point ▪ find the point of intersection between lines ▪ calculate the distance between parallel lines ▪ calculate the angle between two intersecting lines ▪ determine the relative position of a point and a line ▪ calculate the distance from a point to a line ▪ determine the coordinates of the perpendicular projection of a point to a line	*Pupils must be able to and/or understand :* ▪ transform a parametric equation of a line into a cartesian equation ▪ draw, given a point and a line, the parallel line passing through this point ▪ draw, given a point and a line, the perpendicular line passing through this point ▪ find the point of intersection between lines ▪ measure the distance between parallel lines ▪ measure the angle between two intersecting lines ▪ measure the distance from a point to a line ▪ determine the coordinates of the perpendicular projection of a point to a line
Circles in the plane	*Pupils must be able to and/or understand :* ▪ find the equation of a circle ▪ find the equation of the tangent at a point on a circle ▪ relative position of a point and a circle ▪ relative position of a circle and a line ▪ relative position of a circle and a circle ▪ calculate the point(s) of intersection of a line with a	*Pupils must be able to and/or understand :* ▪ explore by constructing, the relative position of a circle and a line ▪ conjecture graphically the equation of the tangent to a circle from a given point ▪ determine using the dot product the equation of the tangent to a circle from a given point ▪ solve simultaneous equations

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	circle: ○ graphically ○ by solving simultaneous equations which can be reduced to a first degree equation and a quadratic equation	