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S6P5 MATHEMATICS SYLLABUS SECONDARY 6th YEAR

Standard level 5 period/week course

APPROVED BY THE JOINT TEACHING COMMITTEE ON THE 4th AND 5TH OF FEBRUARY 2010 IN BRUSSELS

Entry into force in September 2010

ALGEBRA (for guidance: 20 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Introduction of Logarithms :	*Logarithms with a positive integer base should be introduced from powers.* *Pupils must be able to and/or understand:* ▪ the relationship between logarithms and powers. ▪ solve for *x*, equations of the form $a^x = b$, where *a* is a positive integer	*Pupils must be able to and/or understand:* ▪ calculate a logarithm with a positive integer base.
Complex Numbers	*Pupils must be able to and/or understand:* ▪ the concept of complex numbers and the relationship with different sets of numbers. ▪ the real and imaginary parts of a complex number. ▪ the complex conjugate of a complex number. ▪ calculate with complex numbers: sum, product, division and inverse for a non zero complex number. ▪ solve a quadratic equation with real coefficients without a calculator. ▪ solve a equations of the form $z^2 = c$, where c is a complex number.	*Pupils must be able to and/or understand:* ▪ determine the real part, imaginary part, complex conjugate and the inverse of a complex number. ▪ calculate with complex numbers. ▪ to solve a polynomial equation with complex coefficients

ANALYSIS (for guidance: 80 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECHNOLOGY
Sequences	*Pupils must be able to and/or understand:* ▪ the concept of sequences starting from examples. ▪ the notation of the n^{th} term, u_n. ▪ the concept of defining a sequence explicitly and / or recursively. ▪ calculate terms of a sequence using the types of definitions above. ▪ give definitions of arithmetic and geometric sequences. ▪ solve problems involving the properties of arithmetic and geometric sequences : o For example: Finding the first term, common difference, common ratio etc. ▪ calculate a finite series: the sum of n consecutive terms of an arithmetic or geometric sequence and solve problems given the sum. ▪ explore the limit of the n^{th} term of geometric and arithmetic sequences.	*Pupils must be able to and/or understand:* ▪ introduce a sequence with a calculator or software, using both recursive and explicit definitions with and without a spreadsheet. ▪ find the n^{th} term given a sequence. ▪ calculate the sum of consecutive terms. ▪ graph a sequence (n,u_n). ▪ use the advantages provided by a calculator or software to solve more numerically challenging problems.

Note : 1) Except where specified, all the knowledge and skills given in the analysis section apply without a calculator to only this <u>set of basic functions</u> for $a,b,c \in \mathbb{R}$, $\lambda \in \mathbb{R}$.

$$P(x) \text{ polynomial of degree } \leq 3 \text{ ;}$$

$$\frac{P(x)}{Q(x)} \text{ where } P(x) \text{ and } Q(x) \text{ are polynomials of degree } \leq 2 \text{ ;}$$

$$a + \lambda\sqrt{bx+c}; \sqrt{ax^2+bx+c}$$

$$a + \lambda\cos(bx+c); a + \lambda\sin(bx+c); \tan x$$

2) The use of technology throughout this section will enable the study of analysis to not just be limited to the above functions.

TOPIC	KNOWLEDGE & SKILLS	USE OF TECNOLOGY
Introduction to Real Functions	*Pupils must be able to and/or understand:* ▪ determine the domain. ▪ determine both from a graph and algebraically o possible zeros o sign o parity (even, odd functions, or neither) ▪ recognise when a function is periodic and find the period. ▪ the definition of an increasing, decreasing, constant, monotonic function over an interval. As well as the absolute and local extrema. ▪ recognise these characteristics from the graph of a function.	*Pupils must be able to and/or understand:* ▪ draw the graph of a function ▪ choose an appropriate scale and window settings to be able to see all the essential characteristics of the graph. ▪ find algebraically the domain, the zeros, the sign and parity of a function. ▪ change dynamically the graphs of the functions: $x, x^2, \sin x, \cos x, \frac{1}{x}$ to visualise their characteristics and properties. ▪ use the cursor tool to represent and study the graphs of families of functions which has one or more parameters.

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	▪ define the sum, product, quotient and composition of functions. Apply these operations using only the following functions, giving the domain(s) : $x \mapsto x;\ x \mapsto x^2;\ x \mapsto x^3;$ $x \mapsto \frac{1}{x};\ x \mapsto \sqrt{x}$ ▪ sketch the graph of : $x \mapsto x;\ x \mapsto x^2;\ x \mapsto x^3;$ $x \mapsto \frac{1}{x};\ x \mapsto \sqrt{x};$ $x \mapsto \sin x;\ x \mapsto \cos x$ ▪ sketch the graph of : $f(x)+k,\ f(x+k),\ k \cdot f(x),\ f(k \cdot x),$ $\lvert f(x) \rvert,\ f(\lvert x \rvert),\ k \in \mathbb{R}$ where k is a real number and f is one of the following $x \mapsto x;\ x \mapsto x^2;\ x \mapsto x^3;$ $x \mapsto \frac{1}{x};\ x \mapsto \sqrt{x};$ $x \mapsto \sin x;\ x \mapsto \cos x$	
Limits	*Pupils must be able to and/or understand:* ▪ the concepts of finite and infinite limits of a function near a point and infinity. ▪ the concept of limit of a function from the right [resp. left] to a point.	*Pupils must be able to and/or understand:* ▪ determine a limit: two sided limit, one sided limit (from the right and from the left), and limit at an end point of the domain. ▪ use a graph to find the limits above.

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	▪ the theorems of limits, without proof: o of the absolute value of a function, o the product of a function by a real o the sum, product, quotient of two functions o composition of two functions. ▪ calculate all limits for all functions in the set of basic functions.	▪ study functions with one or more parameters, to consider limits, particularly at end points and critical points of the domain.
Indeterminate Forms of the type $"\frac{\infty}{\infty}", "\frac{0}{0}", "0\cdot\infty", "\infty-\infty"$	*Pupils must be able to and/or understand:* ▪ deal with indeterminate forms when finding the limits of all functions in the set of basic functions, as well as : $ax+b+\lambda\sqrt{cx+d};\ \frac{\sqrt{ax^2+bx+c}}{ex+f}$ using the following techniques: simplification, factorisation or division, dominant term, multiplying by a conjugate, using the combined expression, etc.	*Pupils must be able to and/or understand:* ▪ deal algebraically with indeterminate forms when finding the limits of more complex functions, using step by step the following techniques: simplification, factorisation or division, dominant term, multiplying by a conjugate, using the combined expression, etc. ▪ deal graphically with indeterminate forms when finding the limits of more complex functions.
Continuity	*Pupils must be able to and/or understand:* ▪ the concept of continuity of a function from the right [resp. left] to a point. ▪ the theorems of continuity, without proof: o of the absolute value of a function, o the product of a function by a real o the sum, product, quotient of two functions o composition of two functions. ▪ identify a removable discontinuity and redefine the domain.	*Pupils must be able to and/or understand:* ▪ draw a function (also piecewise defined function) and graphically identify the possible discontinuities. ▪ study functions (also piecewise defined function) with one or more parameters, to consider continuity.

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Differentiation	*Pupils must be able to and/or understand:* ▪ that the derivative of a function at a point is defined as the gradient of the tangent at the point. ▪ if the two sided limit of the gradient of a secant exists it will be the derivative of a function at a point. ▪ calculate the derivative at a point from first principles of the following functions : $x \mapsto x ; x \mapsto x^2,\ x \mapsto x^3,\ x \mapsto \frac{1}{x},\ x \mapsto \sqrt{x}$. ▪ Use first principles to find the derivative function for : $x \mapsto x ; x \mapsto x^2,\ x \mapsto x^3,\ x \mapsto \frac{1}{x},\ x \mapsto \sqrt{x}$ ▪ find the equation of the tangent at a point on the graph of a function. ▪ the relation between continuity and differentiability. ▪ the concept of the second and third derivative function. ▪ calculate the first, second and third derivatives of the set of basic functions ▪ apply the rules of differentiation, from the set of basic functions, the following: o the sum o the product o the quotient o the composition (chain rule)	*Pupils must be able to and/or understand:* ▪ construct a tangent to a given curve at a point and determine its gradient and / or equation. ▪ use a spreadsheet to create a table of values of the derivatives at a point and represent these as a scatter plot. ▪ calculate successive derivatives of functions. ▪ represent on the same set of axes a function and its derivatives. ▪ determine from a graph of a function and / or its derivative, whether or not the function is differentiable at a point. ▪ study the possible differentiability of a family set of functions.

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Application of limits and derivatives	*Pupils must be able to and/or understand:* ▪ apply l'Hospital's rule. (without proof) ▪ to apply the concepts of limits and differentiation to study the set of basic functions with regard to: where the graph is increasing and decreasing; finding extrema; finding asymptotes; finding points of inflexion (curvature). ▪ sketch/draw the graph of the function. ▪ deduce only from a given graph all information about the function, related to the topics ''Introduction to real functions'', ''limits and continuity'' and ''differentiation'' ▪ determine the characteristics of a function knowing the graph of its derivative and vice versa.	*Pupils must be able to and/or understand:* ▪ represent a function and investigate its features algebraically ▪ representing sets of families of functions with a slider. ▪ calculate, step by step, a limit using l'Hospital's rule. ▪ examine a given property of a family of functions depending on the parameter. ▪ solve optimisation problems using differentiation. ▪ solve geometry based optimisation problems graphically. ▪ determine the locus of certain points (extrema, inflexion, ...) of a family of functions.

GEOMETRY (for guidance: 30 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECNOLOGY
Geometry in three dimensions	*Pupils must be able to and/or understand:* ▪ the vocabulary and notation of points, lines, planes, spheres. ▪ the relative positions of points, lines, planes, spheres. ▪ the orthogonal projection of a point on a plane, on a line and a line on a plane. ▪ understand the concept of vectors, vector sum, the scalar multiple of a vector, collinear vectors, linear combinations of two vectors, coplanar vectors. ▪ that a line and a plane can be represented by a position vector and a linear combination of direction vectors. ▪ that two vectors create an angle and that two vectors could be orthogonal. ▪ the concept of scalar product of two vectors. ▪ the concept of the length of a vector and the distance between two points. ▪ the concept of the distance (by orthogonal projection) between a point and a plane, a point and a line, and two points in a plane. ▪ the concept of the vector product (cross product) of two vectors.	

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Analytical Geometry	*All these skills must be mastered without a calculator. The aim here is not to evaluate the computational ability of a student but an understanding of spatial geometry.* *Pupils must be able to and/or understand:* ▪ the concept of orthonormal coordinate system. *Throughout the geometry section, the coordinate system chosen will always be orthonormal.* ▪ the concept of coordinates as the components of a position vector and vice versa. ▪ determine whether three vectors are coplanar or not by using a determinant. ▪ the analytical expression of the scalar product (dot product) of two vectors, the length of a vector, the distance between two points. ▪ calculate the coordinates of the perpendicular projection. ▪ calculate the distance of a point to a straight line and of a point to a plane. ▪ find parametric and cartesian equations of a plane. ▪ find parametric and cartesian equations of a straight line. ▪ calculate the angle between two vectors. ▪ calculate the cross product (vector product) of two vectors	*The use of technology allows students to solve geometric problems numerically.* *Pupils must be able to and/or understand:* ▪ solve systems of equations and understand the geometrical meaning of the solutions. ▪ calculate a determinant. ▪ calculate the length of a vector. ▪ calculate the dot product (scalar product) of two vectors ▪ solve problems of analytical geometry requiring extensive calculation (e.g. systems of equations larger than 3x3)

PROBABILITY (for guidance: 20 periods)

TOPIC	KNOWLEDGE & SKILLS	USE OF TECNOLOGY
Combinatorics and Probability	*Pupils must be able to and/or understand:* ▪ count the number of elements in a set using o permutations of a finite set with and without repetitions o combinations of a finite set with and without repetitions $\left[C_n^k = \binom{n}{k} \right]$ ▪ the formulas of combinations : $\binom{n}{0}, \binom{n}{1}, \binom{n}{n}, \binom{n}{k} = \binom{n}{n-k}, \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$ ▪ Pascal's triangle and Newton's binomial theorem $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$ (without proof) ▪ calculate the probability of an event: $P(A) = \frac{n(A)}{n(\Omega)}$	*Pupils must be able to and/or understand:* ▪ calculate the number of elements in a set using permutations and combinations ▪ solve probability problems requiring extensive calculations

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	▪ apply the concepts and skills from the 5th year syllabus o sample space o Venn diagrams o tree diagrams of independent events (sampling with replacement) o tree diagrams of conditional events (sampling without replacement) o the concept of conditional events $P(A \mid B)$ $[P_B(A)]$ o two by two table (cross tabulation) o solve probability problems using: $P(\bar{A}) = 1 - P(A)$ for complementary events $P(A \cap B) = P(A) \cdot P(B)$ for independent events $P(A \cap B) = 0$ for mutually exclusive events $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = 1$ for exhausitive events $P(A \cap B) = P(B) \cdot P(A \mid B)$ $\quad [P(A \cap B) = P(B) \cdot P_B(A)]$ ▪ Bernoulli trial: calculate the probability of k successes in a sequence of n Bernoulli trials with p the probability of success using $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$	