Schola Europaea

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Mathematics Syllabus - Year 1, 2, 3 (Secondary)

APPROVED BY THE BOARD OF GOVERNORS OF THE EUROPEAN SCHOOLS ON 22ND AND 23RD JANUARY IN BRUSSELS

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PREAMBLE

1. OBJECTIVES

The foundations for future mathematical success are laid in the Primary School and in the early years of Secondary School. In the first year of Secondary School, we need to consolidate the learning that has taken place earlier, especially in terms of arithmetical and geometric understanding; 75% of the time should be devoted to this. In the second year of Secondary School, we need to consolidate the learning that has taken place earlier, especially in terms of arithmetical, algebraic, geometric and statistical understanding, while developing the new skills and knowledge necessary for success in mathematics, the sciences and the social sciences. The main themes of the third year syllabus extend those taught during the second year, but presenting the syllabus as a continuation of these subjects in no way indicates that they will then be completed.

The syllabus for first year (secondary) has been based on the harmonised primary syllabus. A copy of this syllabus can be found at **www.eursc.eu**.

1.1. General objectives

The secondary section of the European Schools has the two objectives of providing formal, subject-based education and of encouraging pupils' personal development in a wider social and cultural context. Formal education involves the acquisition of knowledge and understanding, concepts and skills within each subject area. Pupils should learn to describe, interpret, judge and apply their knowledge. Personal development takes place in a range of spiritual, moral, social and cultural contexts. It involves an awareness of appropriate behaviour, understanding of the environment in which pupils work and live and a development of their individual identity.

These two major objectives are inseparably nurtured in the context of an enhanced awareness of the richness of European cultures. This awareness and the experience of a shared European life should lead pupils towards a respect for the traditions of each country in Europe, while preserving their own individual identities.

1.2. Subject-specific objectives

Mathematics instruction must progress systematically and create a lasting foundation for the assimilation of mathematical concepts and structures. The aim is to develop pupils' mathematical skills, such as creative, logical and analytical thinking. Pupils should develop the skills of formulating mathematical problems appropriately, then finding the solutions to the problems and finally presenting their methods and conclusions in a neat and orderly fashion. Problems that come up in day-to-day situations, and that can be resolved with the aid of mathematical thinking or operations, are to be

utilised effectively. Pupils also need to understand and use graphical methods to present information and facilitate understanding. Information and communication technology are to be used to support pupils' learning process.

2. CONTENT

The syllabus, at the end of this document, is mainly presented in three columns. The topic is stated in the first column, the second column determines the objectives to attain and the third column offers some possible teaching approaches. The annotation *'not covered in Primary School'* will state if a topic is new from Primary School. It can therefore be assumed that all other topics will have already been met in Primary School. This should help teachers to ensure a smooth transition from primary to secondary mathematics.

Teachers are free to teach each year's subject matter in any order and in any way they feel appropriate.

3. METHODOLOGY

Many pupils find it difficult to deal with problems expressed in words and link problems to known mathematical techniques in order to solve them. To remedy this situation an investigative approach is strongly recommended. This will greatly help pupils to develop their mathematical knowledge and understanding and to discover mathematical models and structures. This is the reason for the "Problem solving" section in the syllabus.

Teachers should ensure that pupils develop facility with mental calculations.

Even though calculators are part of our everyday environment, it is important to ensure that they are used appropriately. It is recommended that the use of calculators (non-graphical, non-programmable) should be restricted until the students demonstrate facility with traditional "paper and pencil" methods of calculation. The calculator should not be used solely as a calculating aid but, by thoughtful use, help the pupil have a better understanding of the structures behind mathematical operations. That is, it should contribute to developing understanding rather than acquiring techniques.

The teacher should take advantage of any opportunity, arising during lessons, to introduce pupils to algorithmic procedures.

3.1. Problem solving

Problem solving has an important role in mathematical development because it motivates pupils and encourages reasoning skills. Examples and problems can be taken from the real world. In addition, artificial situations, as well as explorations and experiments can be created. In Primary School, students will have already been involved with problem solving in a practical, project based way.

In the context of problem solving, it is desirable that pupils can:

- make use of the usual operations in concrete situations, solve problems using these operations and get to know the appropriate terms associated with each operation
- discover the methods appropriate to solving problems through using equations and inequations in one unknown
- solve problems involving:
    - mathematical puzzles
    - sharing
    - using means
    - using proportionality
        - quantity and price
        - percentages, changes, scales etc.
        - interest
- meet counter-examples
- begin to study problems involving listing
- play games involving combinations
- represent statistical data
- become familiar with elementary notions of probability
- represent situations graphically
- interpret flow-charts.

4. ASSESSMENT OF LEARNING OUTCOMES

4.1. Functions and principles

Assessment is both a formative and a summative process.

Formative assessment is an ongoing process providing information about pupils' learning. It should also be a basis for pupils' further development and it plays an important role in the provision of educational guidance for pupils, parents, or guardians and the school. Assessment need not always involve the

award of a mark and it should not be punitive, but it should evaluate performance. For teachers, assessment of learning outcomes provides an opportunity to review the objectives, methods and results of their teaching.

Summative assessment provides a clear statement of the knowledge and skills possessed by a pupil at a particular point in time.

The following general principles of assessment of learning outcomes should be observed:

- performance should be assessed against all the objectives relating to knowledge and skills set out in the syllabus
- assessment must relate to work which has been covered in the course
- all types of work done by the pupil on the course should be a part of the assessment process - eg. oral and written contributions, class tests, practical work
- pupils should be aware of the work to be done and the standards to be achieved in order to attain each level in the assessment scale
- pupils should know how their performance compares with other pupils, in the same or other sections; this requires co-ordination between the teachers of the same and different sections to ensure comparability.

4.2. Subject-specific assessment

In the first three years of Secondary School, teachers assess pupils' progress twice or three times yearly according to the regulations of the school by awarding a single, whole-number mark. This mark reflects performances in tests and classwork. Homeworks may also be taken into account. Further information on assessment may be obtained from the European Schools' website, which can be found at **www.eursc.eu**.

I. FIRST YEAR SYLLABUS:

I.1 Numbers

- Much of the work will reinforce the work already tackled in the Primary School. Pupils should develop arithmetical skills and become more familiar with rational numbers.
- Mental arithmetic skills, estimation and an understanding of number size should be developed, especially through oral exercises and approximations.
- Calculators will have already been met in limited situations in Primary School. They should now be permitted for checking results and solving more complicated numerical problems
- The objective of this section is not to make a theoretical study of the sets $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}^+$ but to develop ideas and to familiarise pupils with the properties of operations.
- By solving number problems the pupils will begin to recognise the significance of these operations.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
<u>Natural numbers ℕ (including 0)</u>		
Order in ℕ	Order a set of natural numbers and place them on a number line. Work with large numbers. Use the transitivity property of > and <.	Not inequations.
Infinite size of ℕ	Practise written calculations and use mental arithmetic.	
Arithmetical operations		
Long division and long multiplication		4 digit numbers divided by 2 digit numbers, 3 digit numbers multiplied by 3 digit numbers.
Powers of numbers (natural indices only)	Use index notation.	
The importance of 0 and 1 *(not covered in Primary School)*		
Commutative, associative and distributive rules *(not covered in Primary School)*	Know rules of priority and handle brackets.	Pupils are not expected to know these names.
Multiples, factors and prime numbers	Divide by 2, 4, 5, 25, 10, 100, 3 and 9. Determine the prime numbers less than 100. *(not covered in Primary School)*	
Highest Common Factors and	Write a number as a product of prime numbers. *(not covered in Primary School)*	

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Lowest Common Denominator Sets in $\mathbb{N}$	Consider examples of large numbers set in real situations found in other subjects. Read and write large numbers and understand the significance of the position of a digit in a number.	
<u>Integers $\mathbb{Z}$</u>		
Introduction	Place integers on the number line. Compare two integers. Add and subtract integers.	Discuss real situations involving negative numbers. Operations with negative numbers. *(not covered in Primary School)*
<u>Positive rational numbers $\mathbb{Q}^+$</u>		
Rational numbers expressed as decimals and fractions: Comparison of rational numbers Operations on rational numbers	<u>Decimals</u> Revision of how to: a) read and write decimals b) put a set of decimal numbers in order of size and represent them on a number line c) calculate using decimals	Revision and deepening of work done in the Primary School. Emphasis should be put on understanding the significance of the position of a digit in a number. Introduce division by decimal numbers.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
	d) estimate the order of magnitude of an answer e) round numbers (e.g. to one decimal place).	Consider multiplication and division by a decimal, particularly decimals less than 1.
Percentages (only the simplest, i.e. 50%, 25%, 20% and 10%)	<u>Fractions</u> Revision of how to: a) understand fractional notation b) sort fractions into order of size and place them on the number line c) find equivalent fractions d) change a fraction into a decimal and vice-versa e) simplify fractions f) multiply and divide by fractions g) add and subtract simple fractions h) calculate and use simple percentages.	Fractions can be met as: – quotients – an expression of size (the significance of the numerator and denominator) – operators. Try to compare fractions without reducing them to common denominators. Avoid converting fractions into recurring decimals (e.g. 3/7). Use operators to develop arithmetic skills.

I.2 Algebra

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Simple algebraic statements *(not covered in Primary School)*	Read and recognise the operations mentioned and implied in algebraic expressions. Calculate the value of expressions in $\mathbb{Q}^+$ using the hierarchy of operations. Recognise sums, products etc.	
Simple equations	Understand the meaning of and solve in $\mathbb{Q}^+$ equations of the type: $x \pm a = b$ $ax = b$ $ax + b = c$ $\frac{x}{a} = b$ $a,b,c \in \mathbb{Q}^+,\ a \neq 0$	Translation of real situations into equations and inequations in one unknown. Solving equations by using operations without using the principles of equivalence.
Plotting points in planes	Plot points using co-ordinates (coordinates in $\mathbb{N}$). Read information from graphs. Describe situations using sets of points.	Create situations which can be explained as being represented by sets of points or lines or line segments.

I.3 Descriptive statistics

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Collecting and displaying data Representation of data	Collect and display data in – tables – bar charts – histograms. Interpret diagrams.	See chapter 3.1. “Problem solving”. Pupils should be introduced to computer packages for graphing statistical data.
Arithmetic mean	Calculate means.	

I.4 Geometry

Experience has shown that if pupils are to gain a true visual sense of geometry and appreciate objects, shapes and their properties the teaching of geometry must begin with a study of space. To this end it is vital that pupils handle and observe objects in practical situations.

However, geometry is more than a series of observations. An appreciation of the mathematical value of these observations is gained if:

- skills such as evaluation, recording, measuring and manipulation are developed
- pupils make discoveries concerning relationships between properties encountered
- properties of objects are studied systematically and applied in simple cases
- pupils are encouraged to develop a coherent understanding, without in any sense meeting an axiomatic approach to geometry.

An introduction to geometry provides a unique opportunity for pupils to learn the correct use of various precision instruments such as rulers, compasses and set-squares. A pupil gradually learns how to draw precise figures and consequently gains a better appreciation of the properties of the figures drawn.

The use of modern software packages to enhance the pupils' learning is encouraged. For example, good packages are CABRI 2, CABRI 3D, Autograph and EUKLID Dynageo.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
The cube, cuboid, cylinder, sphere, square based pyramid, cone	Recognise and name these solids. Classify these solids according various criteria – faces, edges, vertices – parallel and perpendicular faces and edges – curved or flat faces. Recognise the properties of cubes and cuboids and study their nets. Draw these solids in perspective. *(not covered in Primary School)*	Pupils should increase their knowledge of the two and three dimensional shapes which they have already met in Primary School. Scissors, mirrors, square paper, card, compasses, rulers etc. are the necessary tools. Modern computer software can be very useful (see above). Draw nets that work and some that do not. Complete two-dimensional shapes so that they become nets. Identify points, edges and faces that coincide when nets are folded to make a cuboid. Find the other vertices of a cuboid given 3 vertices. Pick out hidden and visible elements on the representation of a solid. Identify cross-sections of solids. *(not covered in Primary School)*
Quadrilaterals, triangles and circles Polygons of 5, 6, 8, 10 or 12 sides	Recognize, classify and name different shapes with respect to: – parallelism – perpendicularity – equality of sides.	Polygons up to 8 sides have been covered in Primary School.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Constructions	Construct using protractor and set-square: – parallel lines – perpendicular lines – perpendicular bisectors of lines – angles of given size. Construct circles according to given conditions.	– find the distances of a point to a line – find the distance between two parallel lines – find the perpendicular heights of triangles and parallelograms – find medians of a triangle – the symmetry properties of certain figures. Develop the idea of a circle as a set of points equidistant from a fixed point. Examples of the use of compasses: draw circles: – of given radii, passing through a given point – where the centre is on a given line – where the centre is on the circumference of another circle. Geometrical software should also be used here.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Measurements of the dimensions of solids and plane figures	Distinguish between – a segment and its length – an angle and its size – a surface and its area – a solid and its volume. Evaluate and measure lengths and sizes of angles. Find areas by counting unit squares enclosed. Calculate perimeters and areas of squares and rectangles and compound shapes constructed from them. Calculate volumes of cubes and cuboids. *(not covered in Primary School)* Perform conversions between different units of measurement.	By measuring, pupils are led to: – associate numbers with objects and understand the usefulness of conventional units and choose the most appropriate units. – develop ideas of approximation and error in measurement and calculation. Only examples from practical, daily-life situations will have been met in Primary School.

I.5 Set theory

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Sets *(not covered in Primary School)*	Define: set, elements of a set, universal set, empty set, union, intersection and complement of a set. Draw Venn diagrams, place elements in appropriate segments and use Venn diagrams to show logical relationships. Appropriately use the symbols $\in, \notin, \cup, \cap, \subset$.	Simple examples from everyday life can be used, particularly for Venn diagrams. Venn diagrams could be used to show the relationships between quadrilaterals with different properties.

II. SECOND YEAR SYLLABUS:

II.1 Numbers

- Much of the work will reinforce the work already tackled in the first year. Pupils should develop arithmetical skills and become more familiar with the properties of numbers.
- Mental arithmetic skills, estimation and an understanding of number size should be developed, especially through oral exercises and approximations.
- Calculators will have already been met in limited situations. They should now be permitted for checking results and solving more complicated numerical problems.
- The objective of this section is not to make a theoretical study of the sets $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$ but to develop ideas and to familiarise pupils with the properties of operations.
- By solving number problems the pupils will begin to recognise the significance of these operations.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Natural numbers $\mathbb{N}$ (including 0)		
Addition, multiplication; their properties	Recognise the properties of these operations, formulate and use them in mental and written calculations.	The application of these properties provides practise in developing arithmetic skills.
Commutative, associative and distributive laws Subtraction, division	Apply the rules of priority in a sequence of operations and handle brackets.	Show that the properties valid for multiplication and addition are not always applicable for subtraction and division.
Powers	Calculate: $a^m \cdot a^n, (a^m)^n, (a \cdot b)^m,$ $a^m \div a^n,\ m > n,$ (where $m \in \mathbb{N},\ n \in \mathbb{N},\ a \neq 0$)	Approach the formulas through numerical examples.
L.C.M. and H.C.F.	Determine the L.C.M. and H.C.F. by factorising. Express an integer as a product of its prime factors.	Review work from first year. The intersection of sets could be used here.
Integers $\mathbb{Z}$		
The integer, its sign, its absolute value, its additive inverse	Use the following definition: $\lvert a \rvert = a$ if $a > 0$ $\lvert a \rvert = 0$ if $a = 0$ $\lvert a \rvert = -a$ if $a < 0$	There are different ways of introducing and representing whole numbers (coloured numbers, arrows etc.)
Order in $\mathbb{Z}$	Order a set of integers and place them on a number line.	

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Addition and subtraction Conservation of order under addition Conservation of equality under addition		
Multiplication, division Multiplication and equality Multiplication and order Properties of addition and multiplication	Calculate using the rules about signs. Apply in $\mathbb{Z}$ the rules already seen in $\mathbb{N}$. Apply the rules of priority in a sequence of operations and handle brackets. Apply the rules of signs with brackets.	A flow-chart can clarify the hierarchy of the operations.
Powers (natural indices)	Apply the formulas found when working in $\mathbb{N}$.	Use examples of the type: $2^2;\ -2^2;\ (-2)^2;\ -(-2)^2;\ -(-2^2)$ $2^3;\ -2^3;\ (-2)^3;\ -(-2)^3;\ -(-2^3)$

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
<u>Rational numbers $\mathbb{Q}$</u>		
Equivalent fractions, simplification of fractions, irreducible fractions	Apply the rules about signs.	The aim in the 2nd year is to improve arithmetic skills. The following types of example can be used: $\frac{3}{4} = \frac{6}{...} = \frac{...}{-24} = \frac{-9}{...}$ $\frac{-3}{4} = \frac{6}{...} = \frac{...}{-16} = \frac{-12}{...} = \frac{3}{-4} = -\frac{3}{4}$
Fractions and decimals on the number line Nesting decimals and fractions	Change a fraction into a decimal. Change a decimal (with a finite number of digits) into a fraction. Nest a fraction between two numbers in decimal form.	Note the inclusive sets $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$.
Addition, subtraction, multiplication and division using rationals	Perform mental and written calculations with rationals.	Using the L.C.M. is not always the most efficient method. The L.C.M. can be found in simple cases without factorisation.
Direct and indirect proportionality	Use equations of the form $y = kx$ and $y = \frac{k}{x}$	Use many examples from everyday life. Illustrate them graphically.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Percentages	Use percentages in simple examples. Calculate in the following way: $p\%$ of $G = \frac{p}{100} \cdot G$ (or equivalent decimal expression) increase G by $p\% = \left(1 + \frac{p}{100}\right) \cdot G$ decrease G by $p\% = \left(1 - \frac{p}{100}\right) \cdot G$ Perform calculations with interest and compound interest. Relate percentages to proportionality.	A lot of examples of daily life can be used in this context. Simple problems should be calculated mentally, e.g. 50% of 18 pupils or 20% of 180 Euro. For more difficult problems the calculator should be used. E.g. Mr. X has a principle of 500 Euro on his bank account and the rate of interest is 4.5% per year. How much does he have after three years? $(1.045)^3 \cdot 500\,€$ Use percentages to find previous prices etc. E.g. A price has risen by 30% and is now 260€. What was the original price?

II.2 Algebra

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Algebraic expressions	Read and recognize the operations mentioned in algebraic expressions. Calculate the value of algebraic expressions by substitution, using the properties of operations and the rules of priority. Recognise sums and products. Use the rules appropriate to the additive inverse. Expand a single bracket expression such as *k(a+b)*.	Express a sequence of calculations orally and in writing. The priorities of operations can be shown through flow-charts. This can be extended to the products of sums and differences including $(a \pm b)^2$, $(a+b)(a-b)$ and simple cases of factorisation
Equations in one unknown and one degree	Solve equations relative to a given set of elements using the properties of arithmetic. Use a formula to calculate the value of one of its elements.	The equivalence sign ($\Leftrightarrow$) between equations should be used. E.g. Calculate one of the parallel sides of a trapezium given its other parallel side, its area and height.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Simple inequations	Understand the meaning of and solve in $\mathbb{Q}^+$ (positive rational numbers) inequations of the type: $\left.\begin{array}{ll} x \pm a < b & (>) \\ ax < b & (>) \\ \frac{x}{a} < b & (>) \end{array}\right\} (a, b \in \mathbb{Q}^+,\ a \neq 0)$ Write solution sets and show these solutions on the number line.	
Plotting points on a plane	Plot points using coordinates. Interpret graphs. Represent practical situations by sets of points.	Some situations can be represented by sets of isolated points, others by a line of points.

II.3 Descriptive statistics

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Collection and ordering of data	Organise data into frequency tables both grouped (of equal class widths) and ungrouped to form frequency distributions. Construct and interpret statistical diagrams – bar charts – histograms – pie charts.	Extend the work done in the 1st year. Distinguish between qualitative and quantitative data. Use of appropriate computer software packages such as Excel.
Arithmetic mean, median and mode	Calculate the mean, median and mode from sets of data and frequency distributions, but not from grouped frequency distributions.	
Relative frequency	Calculate the relative frequency.	Distinguish clearly between the absolute and relative frequency. Pie charts may be used (restrict to 5 divisions).

II.4 Geometry

Experience has shown that if pupils are to gain a true visual sense of geometry and appreciate objects, shapes and their properties the teaching of geometry must begin with a study of space. To this end it is vital that pupils handle and observe objects in practical situations.

However, geometry is more than a series of observations. An appreciation of the mathematical value of these observations is gained if:

- skills such as evaluation, recording, measuring and manipulation are developed
- pupils make discoveries concerning relationships between properties encountered
- properties of objects are studied systematically and applied in simple cases
- pupils are encouraged to develop a coherent understanding, without in any sense meeting an axiomatic approach to geometry.

An introduction to geometry provides a unique opportunity for pupils to learn the correct use of various precision instruments such as rulers, compasses and set-squares. A pupil gradually learns how to draw precise figures and consequently gains a better appreciation of the properties of the figures drawn.

The use of modern software packages to enhance the pupils' learning is encouraged. For example, good packages are CABRI 2, CABRI 3D, Autograph and EUKLID Dynageo.

SUBJECTS	**KNOWLEDGE & SKILLS** *Pupils should be able to:*	**POSSIBLE TEACHING APPROACHES**
Plane, line, half-line, line segment, point	Express orally and/or in writing that: – the plane, the line are infinite sets of points – every line is an infinite part of a plane – each pair of points defines a unique line – every line segment, every half-line is a proper, yet infinite, subset of a line	Set theory could be used here. Use the environment to find physical examples of planes, lines and points.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Distances (point - point, point - line; parallel lines)	Measure, copy, and compare lengths. State the distance properties of the perpendicular bisector and angle bisector.	Link with coordinate geometry
The circle	Express orally and/or in writing the definition of a circle. Introduce the number π and use it to find the circumference of a circle.	
Reflection	Experiment to show reflection and symmetry. Recognise symmetry. Construct the image of a point and a part of the plane under one of these transformations. Use squared paper and coordinates to find images and discover properties of figures. Recognise invariance. Construct using compasses and ruler – perpendicular bisector – angle bisector.	The notion of an ordered pair formed from the point and its image could be used. Demonstrate transformations on computers using software such as Omnigraph or Autograph.
Angles	Find equal angles using the properties of the above transformations. Construct and copy angles.	Examples using vertically opposite angles, alternate angles etc. can be used.
Plane surfaces		

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
1) Quadrilaterals, triangles	Construct quadrilaterals and triangles meeting given criteria of symmetry. Classify quadrilaterals and triangles according to properties of symmetry. Express orally and/or in writing the definitions of – the perpendicular bisector – the altitude (height) – the angle bisector – the median. Construct these lines using compasses and ruler.	The construction of a circumcircle and inscribed circle is justified here.
2) Areas	Recognise figures which have the same area. Calculate the area of quadrilaterals and triangles. Calculate the areas of circles. Calculate areas of compound shapes made up of the above shapes.	

III. THIRD YEAR SYLLABUS :

III.1 Numbers

It is important that pupils are made aware of the following:

i) The coherence of the number system ($\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$).

ii) The introduction of the additive inverse of a natural number leads to the set $\mathbb{Z}$.

iii) The introduction of the multiplicative inverse of a natural number leads to $\mathbb{Q}$.

iv) As well as being shown that the set of rationals is equal to the set of numbers written in decimal form, some of which have an unlimited but periodic form, the pupils will also meet other numbers which are written as non-terminating, non-periodic decimals.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Decimal form	Convert a fraction into decimal form which may be unlimited and periodic.	Examples: $\frac{1}{9} = 0.11111111... \quad = 0.\dot{1}$ $\frac{12}{99} = 0.121212... \quad = 0.\overline{12}$ $\frac{125}{999} = 0.125125... \quad = 0.\overline{125}$
	Convert a recurring decimal into a fraction.	Examples: $0.00444... = \frac{1}{100} \cdot 0.444... = \frac{1}{100} \cdot \frac{4}{9} = \frac{4}{900} = \frac{1}{225}$ Another method: $3.171717...$ $100x = 317.1717...$ $\underline{-x = -3.1717...}$ $99x = 314$ $\Leftrightarrow x = \frac{314}{99}$

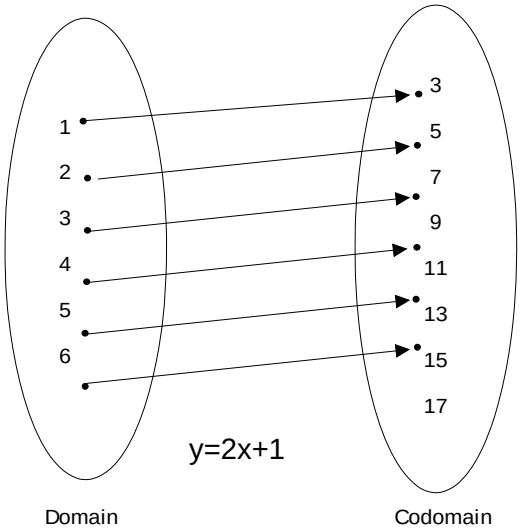
SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
		Consider the question: do non-rational numbers exist, namely numbers which, in decimal form, are non-terminating and non-periodic? E.g.: 0.123456... 1.248163264... 1.357911131517... (where the rule for continuing the digits precludes the existence of a repeating pattern) Likewise, consider the existence of lengths whose magnitude is non-rational. Can these be drawn? Link with Pythagoras.
	Give the most appropriate approximation to a number.	Example: $4.53576\text{ m} \approx 4.536\text{ m}$
Order and bounds – order and addition – order and multiplication – bounds	Replace $a \geq b$ by $a + c \geq b + c$ $a \cdot c \geq b \cdot c$ when $c \geq 0$ $a \cdot c \leq b \cdot c$ when $c \leq 0$ Place a rational number between bounds which: 1° are two consecutive integers 2° get progressively nearer together Estimate the order of magnitude or a result.	Remember that multiplying by a negative quantity changes the order. Use a number line. Example: If $a = \frac{2}{3}$ $a \in [0\,;1]$ $a \in [0.6\,;0.7]$ $a \in [0.66\,;0.67]$ etc.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Operations	Calculate using rational numbers.	
Quotient of two natural numbers (recap)	Determine the (Euclidean) quotient of two numbers. Use results which arise from this relationship.	
Multiplicative inverse of a non-zero rational number	Calculate the inverse of – a number – a product – a quotient	E.g. $a^{-1}, (-a)^{-1}, (a^{-1})^{-1}, (a \cdot b)^{-1}, \left(\frac{a}{b}\right)^{-1}$ $(a, b \neq 0)$
Quotient of two rational numbers	Determine the quotient of two rational numbers correct to one decimal place. Express the quotient exactly in fraction form.	$a \div b = \frac{a}{b} = a \cdot b^{-1} \quad b \neq 0$
Powers (natural indices)	Calculate: $a^m \cdot a^n$ $(a \cdot b)^m$ $(a^m)^n$ $\left(\frac{a}{b}\right)^m$ $\frac{a^m}{a^n}$ $m \in \mathbb{N}, n \in \mathbb{N}$	

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Powers (integers)	Interpret a^n when $n < 0$ Use scientific notation.	Simple cases only Examples : $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $10^{-2} = 0.01$ $0.025 = 2.5 \cdot 10^{-2}$
Ratio and proportion	Recognise and determine quantities which are in – direct proportion – inverse proportion	Extend work covered in second year. Review measures. Numerical tables could be used. Possible applications: $\frac{a}{b} = \frac{c}{d} \Leftrightarrow \frac{a}{c} = \frac{b}{d} \Leftrightarrow a \cdot d = b \cdot c$ This could be extended to: – the sine and cosine ratios – the ratio of the circumference of a circle to its diameter.

III.2 Algebra

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Algebraic expressions	Apply the rules about brackets and simplify an algebraic expression. Calculate the numerical value of an expression by substitution.	
Polynomials	Simplify and order a polynomial in one variable. State the degree of a polynomial. Add, subtract, multiply polynomials in one variable. Use particular products such as $(a \pm b)^2$ $(a+b)(a-b)$	Generalise arithmetic techniques and use them with simple algebraic fractions. Find the degree of a sum or product of polynomials in one variable. Verify this geometrically.
Factorisation	Pick out a common factor in an expression. Factorise expressions such as $a^2 - b^2$ $a^2 \pm 2ab + b^2$	
Algebraic fractions	Use factorisation to simplify algebraic fractions.	Example : $\frac{3x+3y}{x^2+2xy+y^2}$
Substitution	Substitute numbers (particularly negative numbers) accurately into algebraic expressions. Use $f(x)$ notation.	A lot of practice can be given with brackets and powers to avoid later negative sign errors.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Equations, inequations in one unknown and one degree	Solve equations and inequations relative to a given set of elements using the rules of arithmetic. Represent these solutions on a number line. Replace an equation or inequation by an equivalent equation or inequation. Use a formula to calculate the value of one of its elements. Solve problems which involve more than one inequality.	Do not neglect the following cases: $0x < -3$ $0x = 4$ $0x = 0$ etc. Example: calculate one of the parallel sides of a trapezium given its other parallel side, its area and its height.
Relations	Define a relation. Represent a relation graphically using sets and arrows and cartesian graphs.	Define clearly the set of objects, the set of images and the verbal link. 

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Functions Numerical functions of the first degree	Define a function. Establish the domain and range of the function. Represent the ordered pairs of a function on a cartesian graph.	A function can be defined as a special case of a relation. Start from a practical situation. Other functions could be introduced such as $x \to \frac{a}{x}$ $x \to x^2$ and the notations $f(x) = \frac{a}{x}$ $f(x) = x^2$

III.3 Descriptive Statistics

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Probability	Enumerate all the possible results arising from an experiment involving random variables. Enumerate the results fulfilling given conditions. Calculate the probability of an event. Compare this with the relative frequency of an event.	Restrict this work to practical situations.
Collection and ordering of data	Group data in intervals and draw the appropriate histogram (area of column represents relative frequency). Interpret these histograms.	Recall 2nd year work. Show that Excel does not deal properly with histograms. Use practical situations. Example: different histograms having the same mean value.

III.4 Geometry

In the third year, the study of transformations of the plane which was begun in the second year will be extended. The emphasis will be on learning to reason deductively. The geometry course should contribute to a better knowledge of plane figures through discovering their properties, which can then be proved either using transformations or previously known properties. The pupil should be led, through practise, to be able to choose the most suitable method of proof. Although this course does not explicitly mention the geometry of space the teacher should extend concepts about the plane to three dimensions, where appropriate, and also point out where properties which are valid for the plane are not applicable in space.

SUBJECTS	**KNOWLEDGE & SKILLS** *Pupils should be able to:*	**POSSIBLE TEACHING APPROACHES**
Parallel and perpendicular lines	State Euclid's parallel lines postulate in the plane and the theorem about unique perpendicularity, and use them to prove that – if $a \parallel b$ and if $b \parallel c$, then $a \parallel c$ – if $a \parallel b$ and if $b \nparallel c$, then $a \nparallel c$ – if $a \perp b$ and if $b \perp c$, then $a \parallel c$ – etc. Explain why – a line and a plane – two planes are parallel or perpendicular in given situations.	$a \parallel b$ and if $b \parallel c$, then $a \parallel c$ might be the opportunity to introduce – "reductio ad absurdum" – transitivity Counter-examples : if $a \nparallel b$ and $b \nparallel c$, then either a and c intersect or a and c do not intersect if $a \perp b$ and $b \perp c$, then a is not perpendicular to c Limit this topic to an intuitive and explorative approach.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Construction of geometrical figures using compasses and straight-edge only		Consider some open-ended problems Examples: – construct a rhombus given a diagonal and a side – construct a rectangle given a side and the distance between the mid-points of opposite aides (2 cases) – construct a rectangle given a diagonal and one side – etc.
Locus	Define a circle, its interior and exterior. Express using the correct vocabulary the relative positions of a line and a circle.	Construct loci defined by unequal distances from fixed points.
Transformations of the plane: – translations – reflections – rotations – enlargements (positive scale factor)	Define transformations and be aware of their invariants.	Use appropriate computer packages such as Omnigraph or Cabri to show these transformations. Use ruler and compass constructions.
Angles	State and use the following: – vertically opposite angles – alternate angles – corresponding angles – angles either side of perpendiculars – the sum of angles of a triangle and a convex polygon.	Having discovered these properties, they can be justified using transformations of the plane.

SUBJECTS	KNOWLEDGE & SKILLS *Pupils should be able to:*	POSSIBLE TEACHING APPROACHES
Pythagoras' theorem	State and use this theorem and its converse.	This could be justified using the equivalence of areas. Calculate the approximate value of one of the sides of a right-angled triangle knowing the lengths of the other two. Calculate the approximate area of a right-angled triangle knowing the lengths of the hypotenuse and one other side. Given lengths of the sides of a triangle, determine if it is a right-angled triangle.